AN OPTIMIZATION-BASED FRAMEWORK FOR AUTOMATED MARKET MAKING

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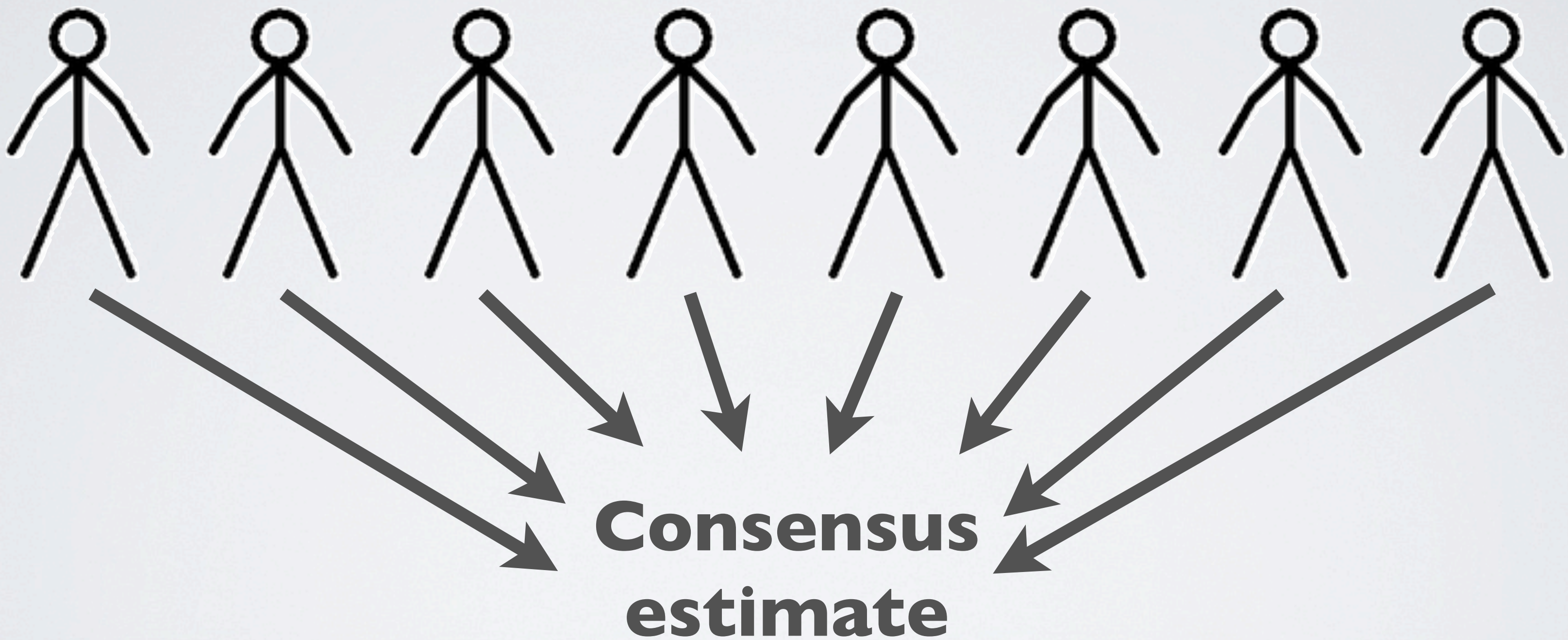
RECAP: PREDICTION MARKETS MERGE BELIEFS



BELIEFS, DISTRIBUTIONS, PRICES

"PROBABILITY DOES NOT EXIST"

- The above phrase is what **Bruno de Finetti** wanted "printed in capital letters in the preface" to his Theory of Probability
- de Finetti: a probability P should be interpreted as the odds of a bet one would offer when our opponent can take any side of this bet.
- The laws of probability can derived via simple "no-arbitrage conditions" of these odds

TWO PROBLEMS FOR AUTOMATED MARKET MAKERS

- As contracts are purchased, how shall we set prices?
- How to handle *combinatorial* outcome spaces, i.e. when N is large?
  - Tournament outcome: $N = n!$
  - Multi-candidate election: $N = \binom{n}{k}$

NAIVE APPROACH: ONE CONTRACT PER OUTCOME

- A natural strategy would just be to sell one contract for each of the large set of outcomes

BETTER APPROACH: A SMALL "MENU" OF CONTRACTS

- Consider a multi-candidate election, where outcome is a set of k winners from n candidates,
- Market maker sells n contracts, one for each i, of the form:

  [pays off $1 when i is among k winners]

- That is, we allow bets on only a subset of "relevant" dimensions

THE PAYOFF MATRIX

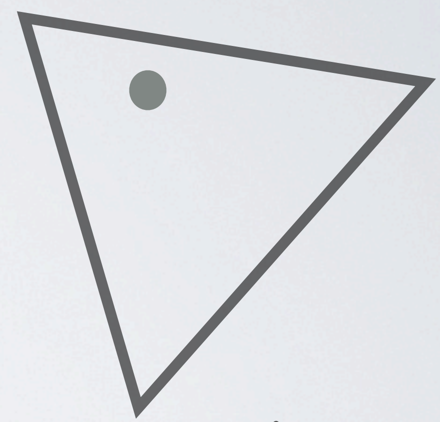
SMALL

L
A
R
G
E

	Cand. 1	Cand. 2	Cand. 3	Cand. 4	Cand. 5	Cand. 6
Outcome 1	$1	$0	$1	$1	$0	$0
Outcome 2	$0	$0	$1	$0	$1	$1
	....					
Outcome N-1	$0	$0	$1	$1	$1	$0
Outcome N	$0	$0	$0	$1	$1	$1

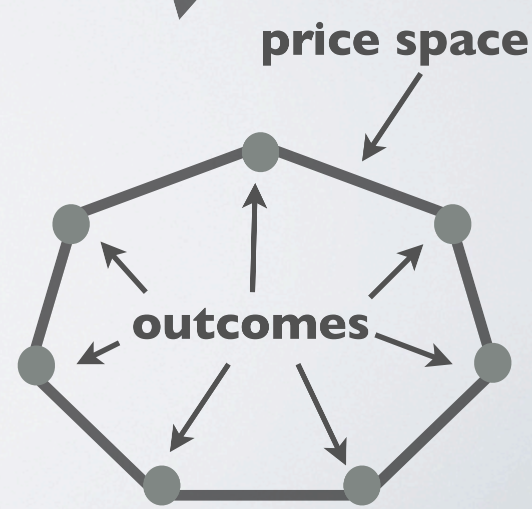
THE PRICE SPACE

- For simple markets, prices lie in the simplex:



- For "complex" markets, what constraints must we impose on the prices?

- Price vector must lie in ConvexHull(outcomes)!



PRICING VIA REGULARIZATION: SIMPLE MARKETS

LMSR: $$\mathbf{p}(i) = \frac{\exp(\eta \mathbf{q}(i))}{\sum_j \exp(\eta \mathbf{q}(j))}$$

Neg. Entropy

Alternative: $$\mathbf{p} := \arg\max_{\mathbf{p}' \in K} \ \mathbf{q} \cdot \mathbf{p}' - \frac{R(\mathbf{p}')}{\eta}$$

Simplex

PRICING VIA REGULARIZATION: COMPLEX MARKETS

Some Curved Regularization

$$\mathbf{p} := \arg\max_{\mathbf{p}' \in K} \; \mathbf{q} \cdot \mathbf{p}' - \frac{R(\mathbf{p}')}{\eta}$$

Convex Hull of Outcome Space

RESULTS

- We have an efficient way to set prices in a prediction market with a combinatorial outcome space
- The "liquidity" (i.e. price stability) depends on the curvature properties of *R* -- more curved => more stability
- The worst-case loss of the market maker is no more than

$$\frac{\max_K R - \min_K R}{\eta}$$

QUESTIONS?

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