

CMPSCI 105: Lecture #8.2 Graphics: Bézier Curves

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Bézier Curves

- A Bézier Curve (named after Pierre Bézier) is:
- A Piecewise,
- Parametric,
- Cubic,
- Polynomial.

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Polynomial

- A simple polynomial is an equation, with:
- An independent variable,
- A series of terms based on powers of that variable,
- Simple numeric coefficients for those terms (no sines and cosines, no calculus, no fancy stuff, just numbers).
- $f(x) = \dots + ax^d + bx^3 + cx^2 + dx + e$

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Cubic

- Maximum power of any term is 3.
- $f(x) = ax^3 + bx^2 + cx + d$
- For Bézier curves this means that there are at most two changes in direction.



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Parametric

- y is not a function of x , locking graph to the coordinate axes, but...
- ...both x and y (and z if we go to 3 dimensions) are now functions of a new independent variable, the parameter, often called t ,
- For Bézier curves this means that the curve can be oriented anywhere in the plane or in space – and is not locked to the coordinate axes.

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Parametric (continued)

- $x(t) = a_x t^3 + b_x t^2 + c_x t + d_x$
- $y(t) = a_y t^3 + b_y t^2 + c_y t + d_y$
- $z(t) = a_z t^3 + b_z t^2 + c_z t + d_z$
- You can extend this method to even higher dimensions, even if you can't visualize the results!

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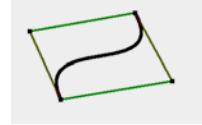
Piecewise

- If you want a complicated shape, you have to stick a bunch of Bézier curves end-to-end, but...
- ...you have to be careful to get one curve to flow smoothly into the next!

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Points

- Four points define a Bézier Curve
 - Two end points and two control points,
 - Each end point associated with one control point.
- The curve goes off infinitely in each direction, but we are interested only in the convex hull:



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Control Lines

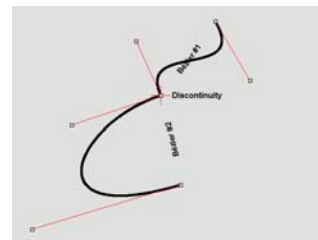
- Each end point and its associated control point form a control line, where...
- ...the Bézier curve is tangent to each control line at the end point



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Joining Two Bézier Curves

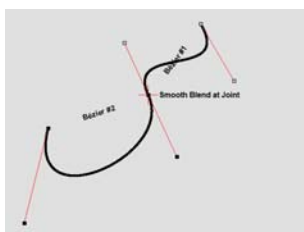
- But control lines don't go same direction:



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Joining Two Bézier Curves

- But control lines DO go same direction:



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Summary

- When joining two Bézier curves end-to-end, make sure that the:
 - Second control point of the first curve,
 - The common end points, and
 - The first control point of the second curve...
- ...are all in a straight line...
- ...and the first curve will blend smoothly into the second!

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