COMPSCI 105: Lecture #9
Graphics: Bézier Curves

Bézier Curves

- A Bézier Curve (named after Pierre Bézier, 1910-1999, engineer at Renault) is:
  - A Piecewise,
  - Parametric,
  - Cubic,
  - Polynomial.

Polynomial

- A simple polynomial is an equation, with:
- An independent variable,
- A series of terms based on powers of that variable,
- Simple numeric coefficients for those terms (no sines and cosines, no calculus, no fancy stuff, just numbers).
- $f(x) = \ldots + \boldsymbol{a}x^4 + \boldsymbol{b}x^3 + \boldsymbol{c}x^2 + \boldsymbol{d}x + \boldsymbol{e}$

Cubic

- Maximum power of any term is 3.
- $f(x) = \boldsymbol{a}x^3 + \boldsymbol{b}x^2 + \boldsymbol{c}x + \boldsymbol{d}$
- For Bézier curves this means that there are at most two changes in direction.


Parametric

- y is not a function of x, locking graph to the coordinate axes, but…
- …both x and y (and z if we go to 3 dimensions) are now functions of a new independent variable, the **parameter**, often called t,
- For Bézier curves this means that the curve can be oriented anywhere in the plane or in space – and is not locked to the coordinate axes.

Parametric (continued)

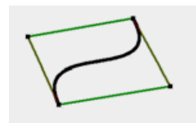
- $x(t) = \boldsymbol{a}_x t^3 + \boldsymbol{b}_x t^2 + \boldsymbol{c}_x t + \boldsymbol{d}_x$
- $y(t) = \boldsymbol{a}_y t^3 + \boldsymbol{b}_y t^2 + \boldsymbol{c}_y t + \boldsymbol{d}_y$
- $z(t) = \boldsymbol{a}_z t^3 + \boldsymbol{b}_z t^2 + \boldsymbol{c}_z t + \boldsymbol{d}_z$
- Where all $\boldsymbol{a}_x$, $\boldsymbol{b}_y$, etc. are just simple numbers.
- You can extend this method to even higher dimensions, even if you can't visualize the results!

Piecewise

- If you want a complicated shape, you have to stick a bunch of Bézier curves end-to-end, but…
- …you have to be careful to get one curve to flow smoothly into the next!

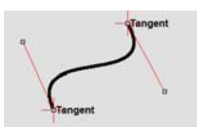
Points

- Four points define a Bézier Curve
  - Two end points and two control points,
  - Each end point associated with one control point.
- The curve goes off infinitely in each direction, but we are interested only in the <u>convex hull</u>:



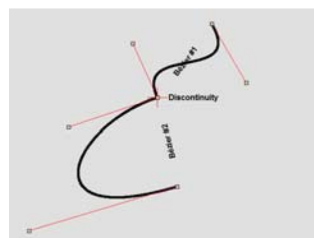
Control Lines

- Each end point and its associated control point form a control line, where…
- …the Bézier curve is tangent to each control line at the end point



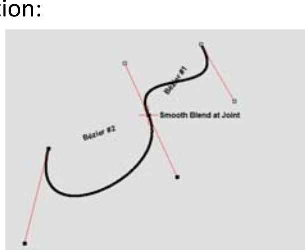
Joining Two Bézier Curves

- But when control lines <u>don't</u> go the same direction:



Joining Two Bézier Curves

- But when control lines <u>DO</u> go the same direction:



Summary

- When joining two Bézier curves end-to-end, make sure that the:
  - <u>Second control point</u> of the first curve,
  - The <u>common end points</u>, and
  - The <u>first control point</u> of the second curve…
- …are all in a **straight line**…
- …and the first curve will blend smoothly into the second!