

## COMPSCI 105: Lecture #4 Gates and Truth Tables

(The Third of Three Math-Heavy Lectures)

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### First, an aside: Binary Addition

- Since numbers can be converted between bases, addition in one base gives the same answer as addition in another base,
- Adding in binary is simpler than adding in decimal,
- So we build computers to add in binary rather than in decimal!

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### Binary Addition Table

+	0	1
0	0	1
1	1	10

Zero, with a Carry

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### Example

• • • • • ← Carries

$$\begin{array}{r} 0110010101000101 \\ +0110011100010110 \\ \hline 1100110001011011 \end{array}$$

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Now back to Logic...

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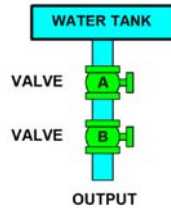
### When is the sentence True?

- **"I am going to the store and going to the beach"**
- Clause A: "going to the store"
- Clause B: "going to the beach"
- Either may independently be True or False.
- When is the overall sentence True?
- Only when **both** are True.

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## When does Water flow?

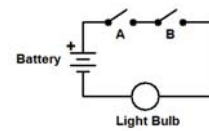
- When both valves are open



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## When does the light come on?

- When both switches are closed



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## Commonalities

- In all three cases the underlying logic is the same:
  - Both clause A and clause B must be True for the sentence to be True.
  - Both Valve A and Valve B must be open for water to flow.
  - Both Switch A and Switch B must be closed for the light bulb to come on.
- The only thing in common is **AND**.

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## Truth Tables

- A **Truth Table** is a tabular way of describing all possible behaviors for a binary system.
- We could use T and F to indicate True and False, but instead we will define 1 for True and 0 for False.
- The number of rows in a Truth Table is 2 raised to the power of the number of inputs.

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## Truth Table for AND (two inputs)

A	B	AND
0	0	0
0	1	0
1	0	0
1	1	1

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## Can we have more than Two inputs?

- YES.**
- "I am going to the store and going to the beach and going to the movies."
- The pipe contains three valves in series, all must be open for water to flow.
- The circuit contain three switches in series, all must be closed for the light bulb to come on.

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### Truth Table for AND (three inputs)

A	B	C	AND
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

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### What About...

- If we can describe the behavior of AND...
- ...can we also describe OR?
- Yes, but there's a catch:
- The OR we use in English is not the same as the OR we use in computer logic!

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### The OR in English

- "I am going to the store OR going to the beach."
- If I don't do either, I've lied,
- if I do one or the other, I've told the truth,
- but *if I do both I've lied!*
- In English the inference is that if I say I'll do one thing or the other, I won't do both!
- This is called an **Exclusive OR** (aka **XOR**).

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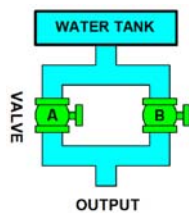
### The OR we use in Logic

- If one input or the other is true, the output is true,
- If both inputs are true, the output is still true,
- This is called an **Inclusive OR**.

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### When does Water flow?

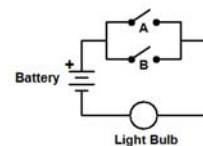
- When either or both valves are open



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### When does the light come on?

- When either or both switches are closed



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### Truth Table(s) for OR

A	B	XOR
0	0	0
0	1	1
1	0	1
1	1	0

A	B	OR
0	0	0
0	1	1
1	0	1
1	1	1

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### General Rules for AND and OR

- AND: The output is 1 if all inputs are 1, the output is 0 if any input is 0.
- OR: The output is 1 if any input is 1, the output is 0 if all inputs are 0.
- XOR: The output is 1 if the inputs differ, the output is 0 if the inputs are the same.

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### NOT

- "I am NOT going to the store."

A	NOT
0	1
1	0

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### Gates

- A Gate is:
  - A **mathematical abstraction** for...
  - ...a **physical device** that...
  - ...implements the function of a **Truth Table**.

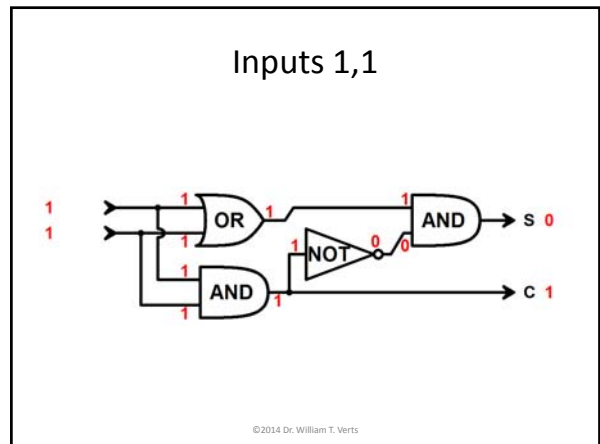
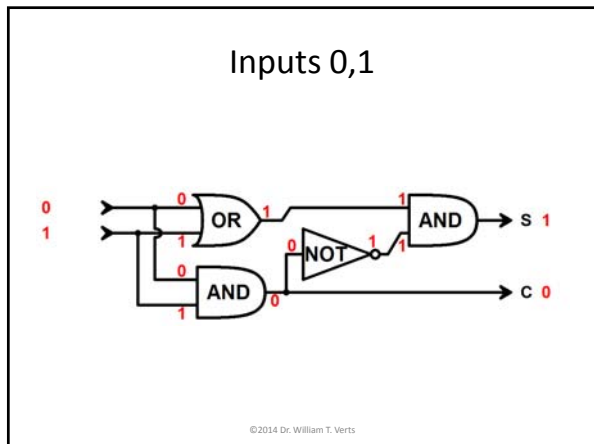
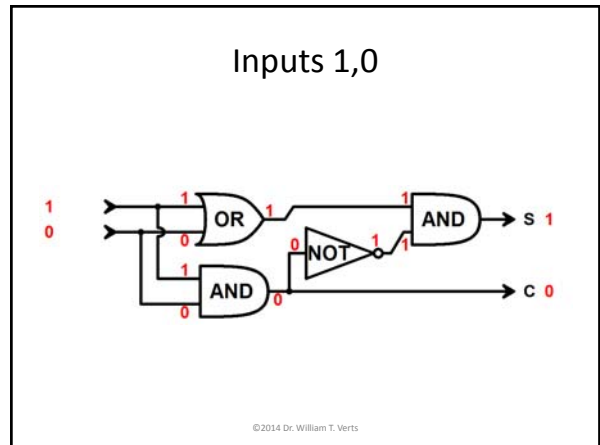
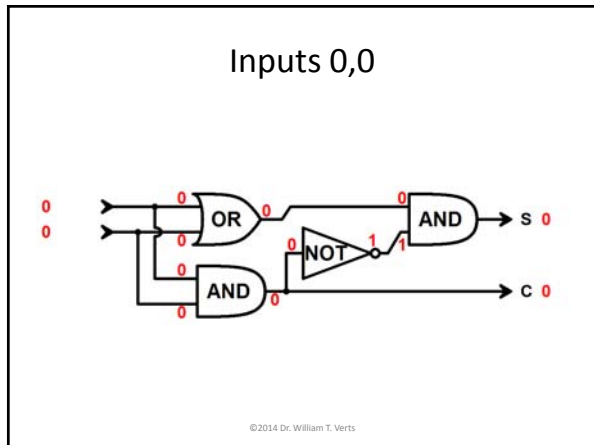
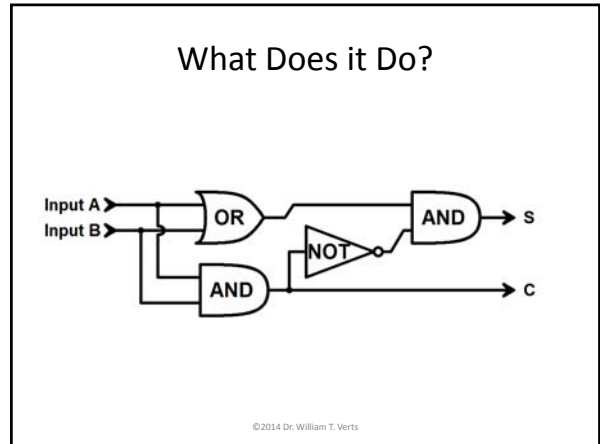
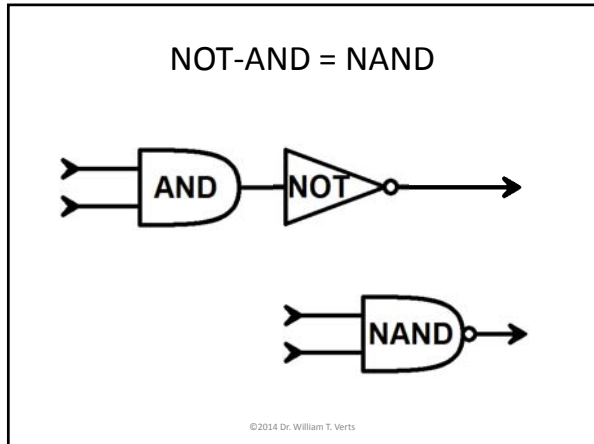
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### AND, OR, and XOR gates

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### NOT-OR = NOR

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What is its Function?

A	B	C	S
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

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A Binary Adder!

A	B	Carry	Sum
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

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