CMPSCI 105: Lecture #4
Gates and Truth Tables

First, an aside: Binary Addition

- Since numbers can be converted between bases, addition in one base gives the same answer as addition in another base,
- Adding in binary is simpler than adding in decimal,
- So we build computers to add in binary rather than in decimal!

Binary Addition Table

+	0	1
0	0	1
1	1	10

Zero, with a Carry

Example

```
 • •   • • •      •      ← Carries
 0110010101000101
+0110011100010110
 1100110001011011
```

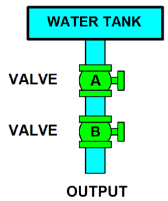

Now back to Logic…

When is the sentence True?

- **"I am <u>going to the store</u> and <u>going to the beach</u>"**
- Clause A: "going to the store"
- Clause B: "going to the beach"
- Either may independently be True or False.
- When is the overall sentence True?
- Only when <u>both</u> are True.

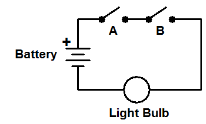
When does Water flow?

- When <u>both</u> valves are open



When does the light come on?

- When <u>both</u> switches are closed



Commonalities

- In all three cases the underlying logic is the same:
  - Both clause A <u>and</u> clause B must be True for the sentence to be True.
  - Both Valve A <u>and</u> Valve B must to open for water to flow.
  - Both Switch A <u>and</u> Switch B must be closed for the light bulb to come on.
- The only thing in common is **AND**.

Truth Tables

- A **<u>Truth Table</u>** is a tabular way of describing all possible behaviors for a binary system.
- We could use T and F to indicate True and False, but instead we will define <u>1 for True</u> and <u>0 for False</u>.
- The number of rows in a Truth Table is 2 raised to the power of the number of inputs.

Truth Table for AND (two inputs)

A	B	AND
0	0	0
0	1	0
1	0	0
1	1	1

Can we have more than Two inputs?

- **YES.**
- "I am <u>going to the store</u> and <u>going to the beach</u> and <u>going to the movies</u>."
- The pipe contains three valves in series, ***<u>all</u>*** must be open for water to flow.
- The circuit contain three switches in series, ***<u>all</u>*** must be closed for the light bulb to come on.

Truth Table for AND (three inputs)

A	B	C	AND
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

What About…

- If we can describe the behavior of AND…
- …can we also describe OR?
- Yes, but there's a catch:
- The OR we use in English is not the same as the OR we use in computer logic!

The OR in English

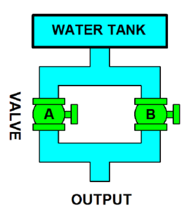
- "I am going to the store OR going to the beach."
- If I don't do either, I've lied,
- if I do one or the other, I've told the truth,
- but *if I do both I've lied*!
- In English the inference is that if I say I'll do one thing or the other, I won't do both!
- This is called an **Exclusive OR** (aka **XOR**).

The OR we use in Logic

- If one input or the other is true, the output is true,
- If both inputs are true, the output is still true,
- This is called an **Inclusive OR**.

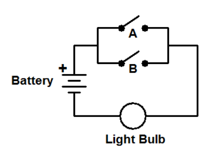
When does Water flow?

- When either or both valves are open



When does the light come on?

- When either or both switches are closed



Truth Table(s) for OR

A	B	XOR
0	0	0
0	1	1
1	0	1
1	1	0

A	B	OR
0	0	0
0	1	1
1	0	1
1	1	1

General Rules for AND and OR

- AND: The output is 1 if all inputs are 1, the output is 0 if any input is 0.
- OR: The output is 1 if any input is 1, the output is 0 if all inputs are 0.
- XOR: The output is 1 if the inputs differ, the output is 0 if the inputs are the same.

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NOT

- "I am NOT going to the store."

A	NOT
0	1
1	0

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Gates

- A Gate is:
  - A **mathematical abstraction** for…
  - …a **physical device** that…
  - …implements the function of a **Truth Table**.

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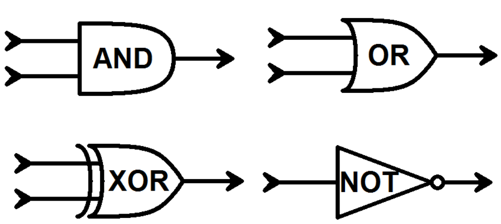
AND, OR, and XOR gates



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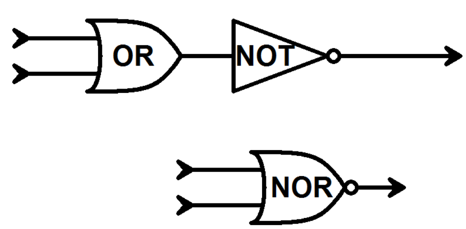
NOT-OR = NOR



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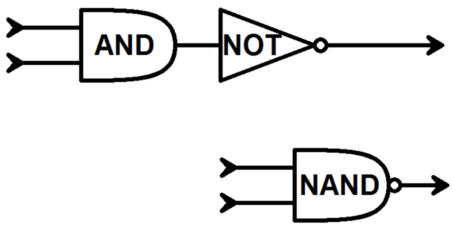
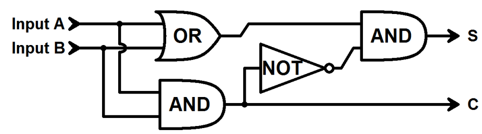
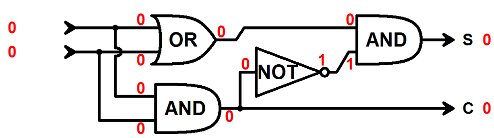
NOT-AND = NAND



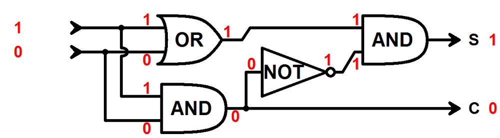
What Does it Do?



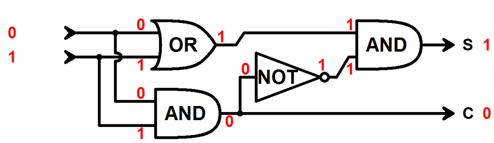
Inputs 0,0



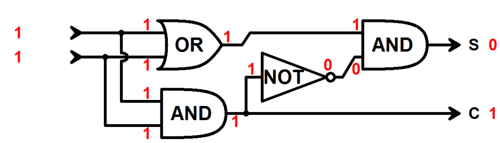
Inputs 1,0



Inputs 0,1



Inputs 1,1



What is its Function?

A	B	C	S
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

A Binary Adder!

A	B	Carry	Sum
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0