COMPSCI 105: Lecture #14
Accuracy vs. Precision
Trusting your Answers

Can you trust your answers?

- It's a computer, right?
- Computers do math, right?
- Spreadsheets do math on computers, right?

- If you've learned anything in this class, you know that computers do only a ***subset*** of math! Remember all that stuff from early in the semester about sizes of integers and floating point? It's all coming back now…

A Refresher about Floating-Point

- Excel uses Double-Precision Floating-Point numbers throughout.
- That means that you have a dynamic range of $\pm 10^{\pm 308}$ but with…
- …only 15-16 digits of precision.
- That means that π is exactly 3.14159265358979 in Excel (it doesn't go on forever), and that numbers bigger than 10^{308} are considered "infinite". Don't like it? Tough.

So what?

- So we have limits on our numbers in Excel,
- Are we ever going to encounter them in practical applications?
- Maybe, maybe not.
- If you do, however, how would you recognize a problem if it occurred?

Example #1

- What do you expect the answer to be?
- 1, right? The **+B3** and the **–B3** cancel.
- In most cases, you would be correct, but…
- If B3 contains 1E100 the answer would be zero!

	A	B	C	D
1				
2				
3		5		
4		=B3+1-B3		
5				
6				

Why?

- When **B3** is 1E100 (1×10^{100}), adding 1 to it doesn't change the value! It's still 1E100.
- There are only 15-16 digits of precision, but to store the 1 I'd need 100 digits. Don't have them, so the 1 vanishes.
- Then subtracting 1E100 from 1E100 gives zero.
- This is called ***catastrophic cancellation***, where very large numbers collide with very small numbers.
- The fix is to rewrite the formula: **=B3-B3+1** to make the large parts more independent of the small parts.

Example #2

- Here's a simple sheet showing formulae on the left and the values on the right.
- Looks OK, right?

	A	B
1		
2		
3		0
4		=B3+0.1
5		=B4+0.1
6		=B5+0.1
7		=B6+0.1
8		=B7+0.1
9		=B8+0.1

	A	B	C	D
1				
2				
3		0		
4		0.1		
5		0.2		
6		0.3		
7		0.4		
8		0.5		
9		0.6		

Look Closer

- Expanding to 20 digits and scrolling down reveals the monster behind the door (it actually started earlier than this):

	A	B
226		22.300000000000000000
227		22.400000000000000000
228		22.500000000000000000
229		22.600000000000010000000
230		22.700000000000010000000
231		22.800000000000010000000
232		22.900000000000010000000
233		23.000000000000010000000
234		23.100000000000010000000

What Happened?

- Why is there noise in the basement of all the numbers?
- The problem is that the number one-tenth ($^1/_{10}$) is finite and well-behaved in decimal, but is an ***infinite repeating fraction*** in binary!
- You can only store numbers with a fixed number of bits, so one-tenth gets truncated.
- The number being added over and over is not one-tenth, but is an ***approximation***.

Round-Off Error

- Doing any calculation with an approximation gives you ***round-off error***,
- Doing the same calculation over and over with an approximation is ***cumulative round-off error***. The error gets bigger, or accumulates.
- As the answer you get diverges from the answer you expect, eventually the error swamps the answer, and you have garbage.

Why do we care?

- Sure, there's round-off error, and it might accumulate, but won't our calculations be good enough?
- Maybe. However:
  - Currency conversion values are mandated by law, and round-off error is not allowed.
  - Scientific calculations may absolutely depend on controlling round-off error.

What can we do?

- One strategy is to keep numbers as integers for as long as possible (instead of $12.52 keep 1252 pennies, for example).
- Perform close analysis of your calculations to determine if round-off error will be a significant problem (it'll always be there, but is it bad enough to worry about?).
- Change the representation of numbers used by spreadsheets (this is a later rant by Dr. Bill).

Paranoid Yet?

- You should be!
- Computers control most of aspects of our lives, but…
- …few programmers and even fewer non-programmers have any notion that these are serious issues that lie at the heart of everything we do.
- There is hope, however…