COMPSCI 145

Representing, Storing, and Retrieving Information

LECTURE #3

CHANGING REPRESENTATIONS

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Why Change Representations?

- Changing representations often means replacing a hard task with an easier one, but
- Doing so may actually make certain other tasks harder.
- Sometimes that doesn't matter!

Back to Arithmetic

- Addition and Subtraction are fairly simple:

```
   1  <---- Carry
  37
+ 49
----
  86
```

- Multiplication is harder:

```
   37
 × 49
 ----
  333
+148
 ----
 1813
```

- Division is significantly worse.

```
      49
   -----
37 | 1813
    -148↓
     ---
      333
     -333
      ---
        0
```


Reduction in Strength

- We can get a speed-up by replacing hard operations with easier ones:
  - Replace $x \div \mathbf{4}$ with $x \times \mathbf{0.25}$ (replace division with multiplication)
  - Replace $\mathbf{2} \times x$ with $x + x$ (replace multiplication with addition)

- This ***reduction in strength*** tends to make complicated operations both simpler and faster.

A Classic Problem

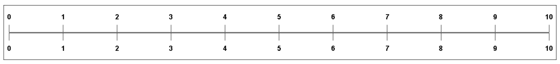
- Back in the 15^{TH} and 16^{TH} Centuries, problems such as computing planetary orbits (Kepler) were all done by hand, and contained a significant number of multiplications.
- If the problems can be recast to exploit a reduction in strength, then the computations can go faster and need not be quite so tedious.
- Can we then recast multiplication as addition?
- YES!

Reduction of Strength on Multiplication

- Here's the initial problem: **N = A × B**
- Take the log of each side: **log(N) = log(A × B)**
- Exploit a property of logs: **log(N) = log(A) + log(B)**
- Take the antilog of each side: **N = antilog(log(A) + log(B))**
- Voila! Multiplication has been replaced by Addition! Reduction in Strength! Problem solved, right?
- But…
- Didn't we just make the problem worse with log and antilog?
- Maybe. Maybe not. Let's investigate…

Take two sticks, with linear scales

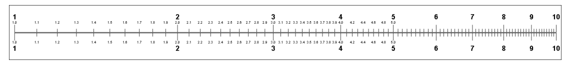
- Start here:



- Move the 0 on the top to above the 3 on the bottom.
  - The 1 is above the 4,
  - The 2 is above the 5,
  - The 3 is above the 6, etc.
- This is obviously an adder. Lengths are added directly.

Now do it with log scales

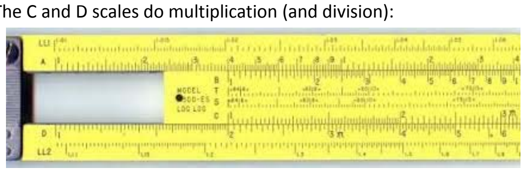
- Start here:



- Move the 1 on the top to above the 3 on the bottom.
  - The 2 is above the 6,
  - The 3 is above the 9, etc.
- We're still adding lengths, but because the scales give us the log and antilog **<u>for free</u>**, we got the reduction in strength we wanted. We can now multiply by adding.

This is a slide rule (from Wikipedia)

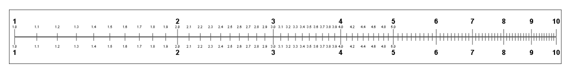
- The C and D scales do multiplication (and division):



- Other scales (front and back) do logs, square root, sine & cosine, etc.

A Problem with Precision

- Start here:



- Move the 1 on the top to above the 2.1 on the bottom.
- Look at the 2.1 on top; what is it above?
- It is near to, but not exactly on, 4.4 (the exact value is 2.1×2.1=4.41)
- This device as shown is only precise to <u>two digits</u>. (Well, it's analog!)