CMPSCI 145 MIDTERM #2

SOLUTION KEY

SPRING 2015

April 3, 2015

Professor William T. Verts

<1> 15 Points – Answer 15 of the following problems (1 point each). Answer more than 15 for extra credit. Incorrect or blank answers will be ignored.

185	What is the decimal value of the 8-bit ***unsigned*** binary number **10111001** ?
-57	What is the decimal value of the 8-bit ***sign and magnitude*** binary number **10111001** ?
-71	What is the decimal value of the 8-bit ***two's complement signed*** binary number **10111001** ?
86	What is the decimal value of the 2-digit ***XS3*** binary number **10111001** ?
0	Circle #1 is at <0,0> with radius 3. Circle #2 is at <0,10> with radius 2. How many points of intersection are there?
1	Circle #1 is at <0,0> with radius 3. Circle #2 is at <0,10> with radius 7. How many points of intersection are there?
2	Circle #1 is at <0,0> with radius 3. Circle #2 is at <0,10> with radius 9. How many points of intersection are there?
0 (#1 inside #2)	Circle #1 is at <0,0> with radius 3. Circle #2 is at <5,5> with radius 30. How many points of intersection are there?
<4,1,0>	A 3D ray has equations $x(t)=6t+1$, $y(t)=-8t+5$, $z(t)=4t-2$. What are the coordinates of the point at $t=½$?
<10,7>	What point in 2D is halfway between <6,4> and <14,10>?
YES	Yes or No: Can I mathematically "draw" a line (i.e., compute intermediate points) between two points in 6 dimensions?
Triangles	Besides spheres, what geometric object is most suitable for ray-tracing?
O(N)	What is the worst case running time (O(1), O($\log_2$(N)), O(N), O(N×$\log_2$(N)), O(N^2), O(N^3), etc.) of a ***linear search***?
O($\log_2$(N))	What is the worst case running time (O(1), O($\log_2$(N)), O(N), O(N×$\log_2$(N)), O(N^2), O(N^3), etc.) of a ***binary search***?
O(N^2)	What is the worst case running time (O(1), O($\log_2$(N)), O(N), O(N×$\log_2$(N)), O(N^2), O(N^3), etc.) of an ***insertion sort***?
O(N×$\log_2$(N))	What is the worst case running time (O(1), O($\log_2$(N)), O(N), O(N×$\log_2$(N)), O(N^2), O(N^3), etc.) of a ***heap sort***?
[9.4, 9.6]	Using interval arithmetic, what is the sum of the two intervals [3.1, 3.2] and [6.3, 6.4] ?
.GIF	Which graphics format(s) (**.BMP**, **.GIF**, **.JPG**, **.PNG**, **.SVG**) supports simple animations?
.JPG, .PNG	Which graphics format(s) (**.BMP**, **.GIF**, **.JPG**, **.PNG**, **.SVG**) supports lossy compression for photographs?
.SVG	Which graphics format(s) (**.BMP**, **.GIF**, **.JPG**, **.PNG**, **.SVG**) supports line-art that can be scaled to any size?

<2> 25 Points – We are going to invent a new floating-point format, called "eighth-precision" which is exactly four bits in length. There is ***no sign bit*** (all numbers are positive), only two bits for the biased exponent, and two bits for the mantissa, but otherwise all rules are the same as single, double, half, and quarter precision. All 16 patterns are shown below, with the exponent field shaded a light gray.

A. (3 Points) What is the value of the bias for this format? $2^{2-1}-1 = \mathbf{1}$

B. (15 Points) Label each pattern to tell me if it is Infinity, NaN, Normalized, Denormal, or Zero (only one answer per pattern). **Three points extra credit** for correctly telling me which NaN patterns are signaling NaN or quiet NaN.

C. (7 Points) For each Normalized pattern, write down its binary scientific notation value, its true binary value, and its decimal value. One pattern is shown with all correct answers. **Three points extra credit** for correctly solving the Denormal patterns as well.

1	1	1	1	NaN (Quiet)					
1	1	1	0	NaN (Quiet)					
1	1	0	1	NaN (Signaling)					
1	1	0	0	Infinity					
1	0	1	1	Normalized	1.11×2^{1}	=	11.1	=	3½
1	0	1	0	Normalized	1.10×2^{1}	=	11.0	=	3
1	0	0	1	Normalized	1.01×2^{1}	=	10.1	=	2½
1	0	0	0	Normalized	1.00×2^{1}	=	10.0	=	2
0	1	1	1	Normalized	1.11×2^{0}	=	1.11	=	1¾
0	1	1	0	Normalized	1.10×2^{0}	=	1.10	=	1½
0	1	0	1	Normalized	1.01×2^{0}	=	1.01	=	1¼
0	1	0	0	Normalized	1.00×2^{0}	=	1.00	=	1
0	0	1	1	DeNormal	0.11×2^{0}	=	0.11	=	¾
0	0	1	0	DeNormal	0.10×2^{0}	=	0.10	=	½
0	0	0	1	DeNormal	0.01×2^{0}	=	0.01	=	¼
0	0	0	0	Zero					

<3> 20 Points – In the list:

April Bob Carol Dan Emma Fred George Hal Iris John Karen Larry Mary Nora Opal

A. (5 Points) Which is the first name examined in a linear search? **April**

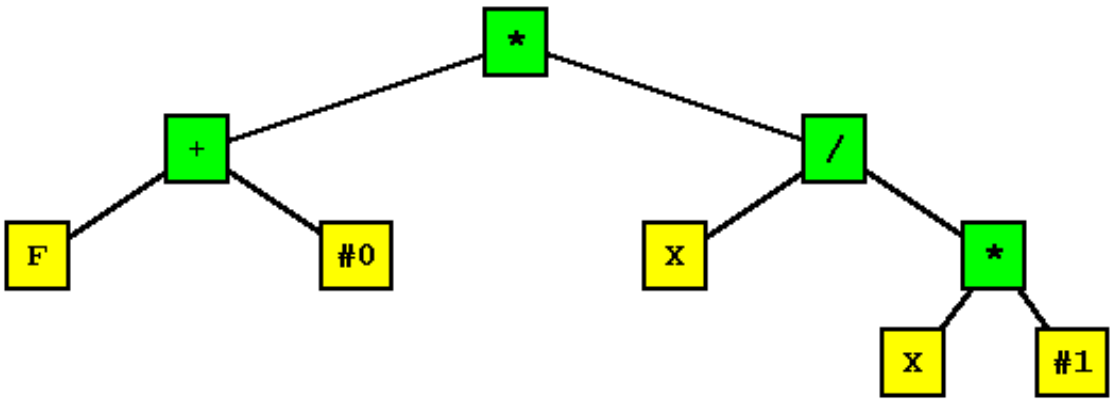
B. (5 Points) Which is the first name examined in a binary search? **Hal**

C. (5 Points) How many names will be examined when looking for Karen using binary search? (Hal, Larry, John, then Karen) **4**

D. (5 Points) In a self-organizing list using the move-to-front approach, what is the order of the names in the list after we search for Fred?

Fred April Bob Carol Dan Emma George Hal Iris John Karen Larry Mary Nora Opal

<4> 10 Points – The image below shows the tree representation for an arithmetic expression (`F` and `X` are algebraic variables, `#0` and `#1` are just the numeric constants 0 and 1, the `*` symbol means multiplication, and `/` means division):



1. (5 points) Write out the complete algebraic equation for the current tree representation ***as shown***, including parentheses where necessary.

```
(F + 0) * (X / (X * 1))
```

2. (5 points) Figure out how to ***optimize*** the tree to most efficiently compute the exact same value as what is shown here, and then draw that resulting tree. (The tree might reduce down to a single node.)

`F + 0` goes to `F`,
`X * 1` goes to `X`,
`X / X` goes to `1`,
`F * 1` goes to `F` (final result). → 

<5> 15 Points – In the following Lagrange interpolation for a parabola, fill in the blanks with numbers so that the curve passes through P_0 when t = -5, through P_1 when t = 0, and through P_2 when t = 5.

$$f(t) = \frac{(t - \mathbf{0})(t - \mathbf{5})}{(\mathbf{-5} - \mathbf{0})(\mathbf{-5} - \mathbf{5})} P_0 + \frac{(t - \mathbf{-5})(t - \mathbf{5})}{(\mathbf{0} - \mathbf{-5})(\mathbf{0} - \mathbf{5})} P_1 + \frac{(t - \mathbf{-5})(t - \mathbf{0})}{(\mathbf{5} - \mathbf{-5})(\mathbf{5} - \mathbf{0})} P_2$$

<6> 15 Points – Short Answer – If someone asked you what a particular binary number "meant," how would you answer them in light of what we have learned in this class? Is it possible to tell them what the binary number actually is, what it can be, or what it cannot be? Justify your answer with examples.

It depends entirely on the **context** in which the number was generated. There is nothing inherent in any binary number which tells us which interpretation to use. We can **discount** some interpretations, such as 1100 cannot be BCD for example.

If we do not know the context, we cannot tell what the number is, but we can say what it can be (list of possibilities for which the number is not illegal), and we can definitely say what the number cannot be.