Biased Normalized Cuts*
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Abstract
We present a modification of “Normalized Cuts” to incorporate priors which can be used for constrained image segmentation. Compared to previous generalizations of “Normalized Cuts” which incorporate constraints, our technique has two advantages. First, we seek solutions which are sufficiently “correlated” with priors which allows us to use noisy top-down information, for example from an object detector. Second, given the spectral solution of the unconstrained problem, the solution of the constrained one can be computed in small additional time, which allows us to run the algorithm in an interactive mode. We compare our algorithm to other graph cut based algorithms and highlight the advantages.

Normalized Cuts for Image Segmentation

Graphs and Laplacians
\[ G = (V, E) \quad \text{where } E \rightarrow \mathbb{R}^k \]

Normalized Cut
\[ \text{Ncut}(S, S^c) = \frac{\sum_{i \in S} \sum_{j \in S^c} w(i, j)}{\text{vol}(S) \text{vol}(S^c)} \]

Spectral Relaxation
\[ \text{min } x^T L_G x \quad \text{subject to: } x^T 1 = 1 \]

Solution using a Spectral Relaxation

Algorithm 1 Biased Normalized Cuts (G, w, s, y, γ)

Require: Graph G = (V, E), edge weight function w, seed s, y and a correlation parameter γ ∈ (−∞, λ2(G))
1. \( A_G(i, j) = w(i, j) \)
2. \( D_G(i) = \sum_j w(i, j) \)
3. \( L_G = D_G - A_G \)
4. Compute \( u_1, u_2, \ldots, u_K \) the eigenvectors of \( L_G \) corresponding to the \( K \) smallest eigenvalues \( \lambda_1, \lambda_2, \ldots, \lambda_K \).
5. \( \text{Minimize } x^T L_G x \quad \text{subject to: } x^T 1 = 1 \)

1. The solution is a weighted combination of the eigenvectors. The weights of the eigenvectors are proportional to the correlation with the seed vector, i.e. eigenvectors that are well correlated get up-weighted.
2. Steps 1-3 are the steps for solving Normalized Cuts.
3. In an interactive setting, only Steps 4-5 need to be repeated.
4. On natural images, eigenvalues grow quickly, so using top K eigenvectors are enough for a good approximation. We set \( K = 25 \) in our experiments.
5. Matrix L, D are sparse so, complexity is linear in the number of pixels.

Comparison of Biased and Constrained Normalized Cuts

Effect of outliers and number of constraints
Adding more constraints improve the Dirak solution while making the Diak solution worse. Entering the constraints in a hard manner makes the desired grouping infeasible.

Biased NCuts for various seed sets T

Effect of Bias and Correlation Parameter (γ)

Top Down & Bottom Up Segmentation

Seed vectors using an object detector

*This work was supported by a Google Graduate Fellowship and ONR MURI N00014-06-1-0734