CS 312: Algorithms **Fall 2018**

Fourth Hour 11

Your Name: _____ Collaborators: _____

You will be randomly assigned groups to work on these problems in discussion section.

Problem 1. Interval Scheduling. For each of the two questions below, decide whether the answer is (i) "Yes", (ii) "No" or (iii) "Unknown, because it would resolve the question of whether P = NP". Explain your answer.

1. Let's define the decision version of the Interval Scheduling Problem from Chapter 4 as follows: Given a collection of intervals on a time-line, and a bound k, does the collection contain a subset of nonoverlapping intervals of size at least k?

   Question: Is it the case that Interval Scheduling $\leq_P$ Vertex Cover?

2. Question: Is it the case that Independent Set $\leq_P$ Interval Scheduling

Problem 2. Diverse Subset A store trying to analyze the behavior of its customers will often maintain a two-dimensional array A, where the rows correspond to its customers and the columns correspond to the products it sells. The entry $A[i, j]$ specifies the quantity of product j that has been purchased by customer i.

Heres a tiny example of such an array A.

	detergent	beer	diapers	cat litter
Raj	0	6	0	3
Alanis	2	3	0	0
Chelsea	0	0	0	7

One thing that a store might want to do with this data is the following. Let us say that a subset S of the customers is *diverse* if no two of the of the customers in S have ever bought the same product (i.e., for each product, at most one of the customers in S has ever bought it). A diverse set of customers can be useful, for example, as a target pool for market research.

We can now define the Diverse-Subset Problem as follows: Given an $m \times n$ array A as defined above, and a number $k \leq m$, is there a subset of at least k of customers that is diverse?

Give a reduction that shows that Diverse-Subset is "at least as hard as" Independent-Set.

(After Monday, we'll understand this as a proof that Diverse-Subset is NP-complete.)