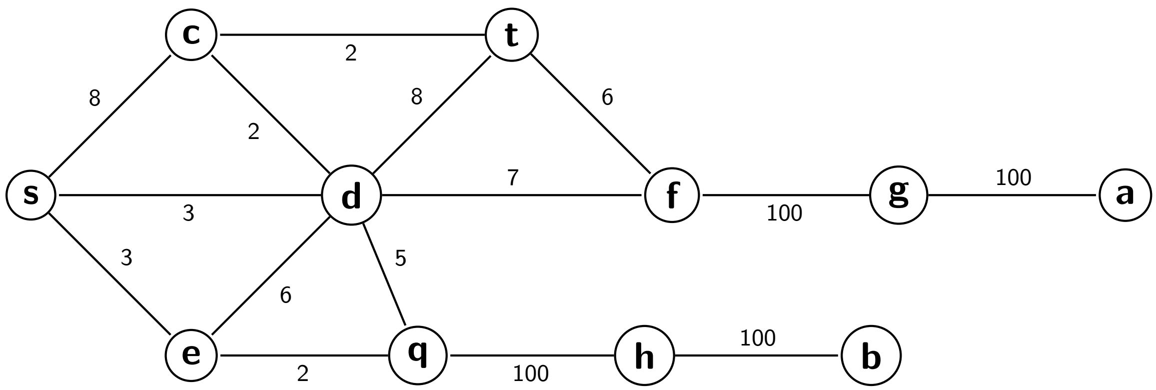
CS 312: Algorithms	**Fall 2018**

Fourth Hour 7

Your Name: ____________________ Collaborators: ____

You will be randomly assigned groups to work on these problems in discussion section.

Problem 1. Dijkstra. Consider this weighted graph.



(a) Execute Dijkstra's algorithm to find a shortest path from node s to rest of the nodes. (Assume each undirected edge corresponds to two directed edges of the same weight pointing in opposite directions.)

(b) Draw an edge between a and b with a weight of -1000. Is there such a thing as a shortest path between s and t in our new graph?

Problem 2. Minimum Spanning Tree. Consider the minimum spanning tree (MST) problem on an undirected graph $G = (V, E)$, with cost $c_e \geq 0$ assigned to each edge e. If the costs are not all distinct, there may be more than one MST.

(a) *Draw an example graph with more than one MST. Identify the MSTs.*

(b) Suppose we are given a spanning tree T with the guarantee that for every edge e in T, that e belongs to *some* MST of G. Can we conclude that T itself must be an MST? Give a proof or a counterexample.

Problem 3. A Puzzle (for fun).

You are to cut out some pieces of paper.

You must be able to place your pieces of paper on an 8x8 checkerboard so that they exactly cover the checkerboard, minus *any* one square.

That is, once your pieces are cut, if I identify to you *any* of the squares on the checkerboard, you must be able to arrange all your pieces of paper (flat, not folded) on the checkerboard so that:

- the pieces of paper don't overlap
- the pieces of paper don't cover any area outside the checkerboard
- the pieces of paper cover all of the checkerboard except the square I chose

The problem: figure out how to do this with three pieces of paper.

For example, it is easy to see how to do this with 63 pieces of paper — use 63 1x1 squares of paper. Or, 32 pieces — use a 4x8 piece and 31 1x1 pieces...

Problem 4. Another Puzzle (for more fun). A real number is assigned to each vertex of a finite connected graph so that the number on any vertex is the arithmetic mean (average) of the numbers on neighboring vertices. Prove that all vertices' numbers are equal.