CS 312: Algorithms **Fall 2018**

Homework 9

Your Name: _____

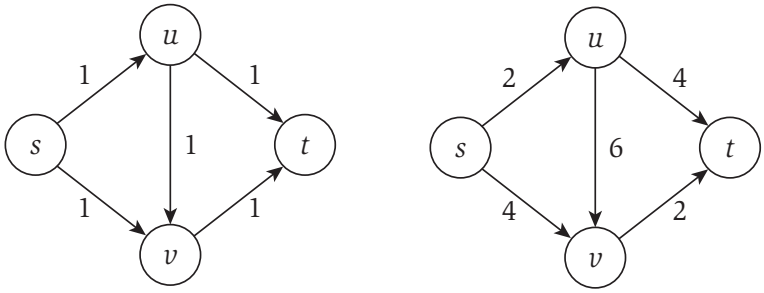
Collaborators and sources: _____

You may work in groups, but you must write solutions yourself. List collaborators on your submission.

If you are asked to design an algorithm, please provide: (a) either pseudocode or a precise English description of the algorithm, (b) an explanation of the intuition for the algorithm, (c) a proof of correctness, (d) the running time of your algorithm and (e) justification for your running time analysis.

Submission instructions. This assignment is due by noon on Thursday, November 29 in Gradescope (as a pdf file). Please review the course policies on the course home page about Gradescope submissions.

1. **(10 points)** *(Work independently.)* K&T Chapter 7, Exercise 1

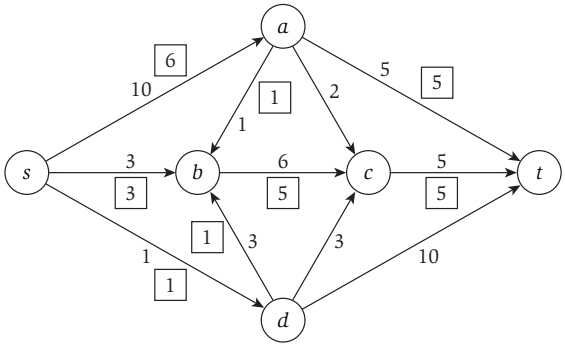


(a) (b)

   (a) List all the minimum s-t cuts in flow network (a) above. The capacity of each edge appears as a label next to the edge.

   (b) What is the minimum capacity of an s-t cut in flow network (b) above? Again, the capacity of each edge appears as a label next to the edge.

2. **(10 points)** *(Work independently.)* K&T Chapter 7, Exercise 3. The following figure shows a flow network on which an s-t flow has been computed. The capacity of each edge appears as a label next to the edge, and the numbers in boxes give the amount of flow sent on each edge. (Edges without boxed numbers have no flow being sent on them.)



(a) What is the value of the flow? Is this a maximum (s,t) flow in this graph?

(b) Find a minimum s-t cut and also say what its capacity is.

3. **(5 points)** *(Work independently.)* K&T Chapter 7, Exercise 4. Decide whether you think the following statement is true or false. If it is true, give a short explanation. If it is false, give a counterexample.

   Let G be an arbitrary flow network, with a source s, a sink t, and a positive integer capacity c_e on every edge e. If f is a maximum s-t flow in G, then f saturates every edge out of s with flow (i.e., for all edges e out of s, we have $f(e) = c_e$).

4. **(5 points)** *(Work independently.)* K&T Chapter 7, Exercise 5. Decide whether you think the following statement is true or false. If it is true, give a short explanation. If it is false, give a counterexample.

   Let G be an arbitrary flow network, with a source s, a sink t, and a positive integer capacity c_e on every edge e; and let (A, B) be a minimum s-t cut with respect to these capacities $c_e : e \in E$. Now suppose we add 1 to every capacity; then (A, B) is still a minimum s-t cut with respect to these new capacities $\{1 + c_e : e \in E\}$.

5. **(10 points)** K&T Chapter 7, Exercise 7. For this problem, you should describe precisely how you solve it but do not need to give a proof of correctness. [This problem is related to bipartite matching, which we will finish in class on Monday]