Problem 1. (3-COLORING and SAT) Prove that 3-COLORING $\leq_P$ SAT. (Hint: use variables to represent the color of a node; encode the constraints that: (1) every node gets at least one color, (2) every node gets at most one color, (3) no edge has two endpoints of the same color.)

Problem 2. (K&T Chapter 8, Example 8.19)

1. Prove that Hamiltonian Cycle $\leq_P$ Hamiltonian Path.
2. Prove that Hamiltonian Path $\leq_P$ Hamiltonian Cycle.
Problem 3. In the PARTITION problem, you are given positive integers $x_1, x_2, \cdots, x_n$ and want to decide whether the numbers can be partitioned into two sets $S_1$ and $S_2$ with the same sum, i.e., such that

\[ \sum_{x_i \in S_1} x_i = \sum_{x_j \in S_2} x_j. \]

Show that PARTITION is NP-complete. (Hint: try a reduction from SUBSETSUM.)