Discussion 3

Your Name: _____   Collaborators: _____

You will be randomly assigned groups to work on these problems in discussion section. List your group members on your worksheet and turn it in at the end of class.

Problem 1. Topological Sorting.

Below is a list of edges in a directed graph with nodes A, B, C, D, E, and F.

$$B \to E, B \to F, C \to D, D \to A, E \to F.$$

1. Find five topological sortings of the graph above.

2. Say that you want the ordering $ABCDEF$ to be a topological ordering. Which edge must be removed to do this?

3. Say that you didn't want any topological orderings to be possible. What is an edge you could add that would achieve this effect.

4. Say that the only nodes in the graph were B, E, and F. How many topological orderings are possible here?

Problem 2. Graphs.

Suppose that an n-node undirected graph $G = (V, E)$ contains two nodes s and t such that the distance between s and t is strictly greater than $n/2$. Show that there must exist some node v, not equal to either s or t, such that deleting v from G destroys all $s - t$ paths. (In other words, the graph obtained from G by deleting v contains no path from s to t). Give an algorithm with running time $\mathcal{O}(m+n)$ to find such a node v.

Problem 3. Graphs.

Graphs. Say that you have a datastructure R. Similar to a stack or a queue, you can put items into R. However, when you pop or dequeue from R, a random item will come out, rather than the item from the front or the back of R.

Similar to how BFS uses queues and DFS uses stacks, you could implement RFS using R.

(a) Say that graph C is connected. If RFS is run on C, will it traverse the whole graph? Prove your answer.

(b) What will the tree produced by RFS be like? Bushy? Scraggly? Tall? Short? Explain.