CMPSCI 311: Introduction to Algorithms **Fall 2022**

Discussion 1

Your Name: _____ Collaborators: _____

You will be randomly assigned groups to complete these problems. List your group members on your worksheet and submit to Gradescope at the end of class.

Problem 1. Stable Matching. Consider the following stable matching instance.

College	Preferences	Student	Preferences
A	$1 > 2 > 3 > 4$	1	$D > B > C > A$
B	$2 > 1 > 4 > 3$	2	$A > D > B > C$
C	$1 > 3 > 2 > 4$	3	$A > B > C > D$
D	$2 > 1 > 3 > 4$	4	$D > C > A > B$

1. Run the Propose and Reject Algorithm on the following instance.
2. Find another stable matching in this instance.

Problem 2. Reading Algorithms Questions Precisely. Consider the statement in the following question (K&T Ch.1 Ex.1):

True or false? In every instance of the Stable Matching Problem, there is a stable matching containing pair (c, s) such that c is ranked first on the preference list of s and s is ranked first on the preference list of c.

1. Describe in words what an "instance of the Stable Matching Problem" is.

2. Let I represent an instance and let M represent a stable matching. Which of the following is a correct logical representation of the statement above?

   (a) $\forall I\ \exists M\ \forall (c,s) \in M\ \text{first}(c,s) \wedge \text{first}(s,c)$
   (b) $\forall I\ \exists M\ \exists (c,s) \in M\ \text{first}(c,s) \wedge \text{first}(s,c)$
   (c) $\forall I\ \forall M\ \exists (c,s) \in M\ \text{first}(c,s) \wedge \text{first}(s,c)$
   (d) $\exists I\ \exists M\ \forall (c,s) \in M\ \text{first}(c,s) \wedge \text{first}(s,c)$

3. Underline the English phrases in the statement that act as quantifiers.

4. Which of the following are correct negations of the logical formula? There may be more than one.

   (a) $\exists I\ \forall M\ \neg\big(\exists (c,s) \in M\ \text{first}(c,s) \wedge \text{first}(s,c)\big)$
   (b) $\forall I\ \exists M\ \forall (c,s) \in M\ \neg\big(\text{first}(c,s) \wedge \text{first}(s,c)\big)$
   (c) $\exists I\ \forall M\ \forall (c,s) \in M\ \neg\big(\text{first}(c,s) \wedge \text{first}(s,c)\big)$
   (d) $\exists I\ \exists M\ \forall (c,s) \in M\ \neg\big(\text{first}(c,s) \wedge \text{first}(s,c)\big)$

5. Write an English statement that is the correct negation of the statement above. Underline the phrases that act as quantifiers.

6. Here is an answer with incorrect reasoning for the original true or false question. Identify exactly what is wrong with the reasoning.

   "The statement is false. Consider the instance:

$c_1 : s_1 > s_2$	$s_1 : c_2 > c_1$
$c_2 : s_2 > s_1$	$s_2 : c_1 > c_2$

   In the stable matching $M = \{(c_1, s_1), (c_2, s_2)\}$, neither student has their first choice, so M does not include a pair (c, s) where c is ranked first on the preference list of s and s is ranked first on the preference list of c."

7. Prove that the original statement is false (i.e., prove that its negation is true).

Problem 3. Big-O Exercises

1. Let $T(n) = 4n^3 + 2n^2 + 2$. Prove that $T(n)$ is $O(n^3)$ using the definition of big-O.
2. Let $T(n) = n \log_2(n)$. It's easy to see that $T(n)$ is $O(n^2)$. In fact, $T(n) = n \log_2(n) \leq n^2$ for all $n \geq 0$. Prove that $T(n)$ *is not* $O(n)$. Try a proof by contradiction.