CMPSCI 311: Introduction to Algorithms **Spring 2017**

Discussion 1

Your Name: ________________ Collaborators: ______________________________

You will be randomly assigned groups to work on these problems in discussion section. List your group members on your worksheet and turn it in at the end of class.

Problem 1. Stable Matching. Consider the following stable matching instance.

College	Preferences	Student	Preferences
A	$1 > 2 > 3 > 4$	1	$D > B > C > A$
B	$2 > 1 > 4 > 3$	2	$A > D > B > C$
C	$1 > 3 > 2 > 4$	3	$A > B > C > D$
D	$2 > 1 > 3 > 4$	4	$D > C > A > B$

1. Run the Propose and Reject Algorithm on the following instance.
2. Find another stable matching in this instance.

Problem 2. Reading Algorithms Questions Precisely. Consider the statement in the following question (K&T Ch.1 Ex.1):

True or false? In every instance of the Stable Matching Problem, there is a stable matching containing pair (m, w) such that m is ranked first on the preference list of w and w is ranked first on the preference list of m.

1. Let I represent an instance and let M represent a stable matching. Which of the following is a correct logical representation of the statement above?

   (a) $\forall I\ \exists M\ \forall(m, w) \in M\ \text{first}(m, w) \wedge \text{first}(w, m)$
   (b) $\forall I\ \exists M\ \exists(m, w) \in M\ \text{first}(m, w) \wedge \text{first}(w, m)$
   (c) $\forall I\ \forall M\ \exists(m, w) \in M\ \text{first}(m, w) \wedge \text{first}(w, m)$
   (d) $\exists I\ \exists M\ \forall(m, w) \in M\ \text{first}(m, w) \wedge \text{first}(w, m)$

2. Underline the English phrases in the statement that act as quantifiers.

3. Which of the following are correct negations of the logical formula? There may be more than one.

   (a) $\exists I\ \forall M\ \neg\big(\exists(m, w) \in M\ \text{first}(m, w) \wedge \text{first}(w, m)\big)$
   (b) $\forall I\ \exists M\ \forall(m, w) \in M\ \neg\big(\text{first}(m, w) \wedge \text{first}(w, m)\big)$
   (c) $\exists I\ \forall M\ \forall(m, w) \in M\ \neg\big(\text{first}(m, w) \wedge \text{first}(w, m)\big)$
   (d) $\exists I\ \exists M\ \forall(m, w) \in M\ \neg\big(\text{first}(m, w) \wedge \text{first}(w, m)\big)$

4. Write an English statement that is the correct negation of the statement above. Underline the phrases that act as quantifiers.

5. Here is an answer with incorrect reasoning for the original true or false question. Identify exactly what is wrong with reasoning.

   "The statement is false. Consider the instance:

   $$\begin{array}{l|l} m_1 : w_1 > w_2 & w_1 : m_2 > m_1 \\ m_2 : w_2 > w_1 & w_2 : m_1 > m_2 \end{array}$$

   In the stable matching $M = \{(m_1, w_1), (m_2, w_2)\}$, neither woman has her first choice, so M does not include a pair (m, w) where m is ranked first on the preference list of w and w is ranked first on the preference list of m."

6. Prove that the original statement is false (i.e., prove that its negation is true).

Problem 3. Proofs in English, Proof by Contradiction.

1. Read this problem (Solved Exercise 1 from K&T—don't read the solution until after class). Don't solve it until after you read the next section.

   Consider a town with n men and n women seeking to get married to one another. Each man has a preference list that ranks all the women, and each woman has a preference list that ranks all the men.

   The set of all $2n$ people is divided into two categories: *good* people and *bad* people. Suppose that for some number k, $1 \leq k \leq n-1$, there are k good men and k good women; thus there are $n-k$ bad men and $n-k$ bad women.

   Everyone would rather marry any good person than any bad person. Formally, each preference list has the property that it ranks each good person of the opposite gender higher than each bad person of the opposite gender: its first k entries are the good people (of the opposite gender) in some order, and its next $n-k$ are the bad people (of the opposite gender) in some order.

   Show that the following statement is true:

   A: In every stable matching, every good man is married to a good woman.

2. Read this and discuss with your group members:

   What is a proof? A proof begins with a set of known facts and reasons through logical deductions from these to prove another fact. Each step of the proof should follow very clearly from the previous steps. In this class, proofs will usually be written in English. This may be new to you. It is important to recognize that proofs in English still follow the structure of more symbolic proofs: they reason very precisely from an established set of facts to another one through precise statements. *It is important to learn to understand and write precise logical arguments in English.*

   The set of facts you may use in a proof are:

   - Assumptions given in the problem setup (e.g., the assumption that all good men are ranked ahead of bad men, etc.)
   - Definitions (e.g., the definition of a stable matching)
   - Basic mathematical facts (e.g., if one good man is married to a bad woman, there are only $n-1$ good men remaining)
   - Any common knowledge in the context of whoever will be reading your proof. In this class, common knowledge includes facts proven or asserted in lectures, homework, or the textbook. Use these, with reference if it is not obvious.

   There are different styles of proof.

   Lets consider proving statement A. It begins with a "for all" quantifier: every stable matching has a certain property. In this situation, a proof by contradiction is often best. We (temporarily) suppose that A is false, and then prove the negation of some of other fact we know is true. This implies that our supposition was wrong, and so A must actually be true. An advantage is that we start by negating A, which switches the "for all" quantifier to a "there exists" quantifer:

   $\neg A$: There exists a stable matching where not every good man is married to a good woman.

   We now get to reason about a *specific* stable matching instead of *all* stable matchings.

3. Now, prove that A is true by contradiction.