Instructions. You may work in groups, but you must write solutions yourself. List collaborators on your submission.

If you are asked to design an algorithm, please provide: (a) the pseudocode or precise description in words of the algorithm, (b) an explanation of the intuition for the algorithm, (c) a proof of correctness, (d) the running time of your algorithm and (e) justification for your running time analysis.

Submissions. Please submit a PDF file. You may submit a scanned handwritten document, but a typed submission is preferred. Please assign pages to questions in Gradescope.

1. (10 points) Flows and Cuts. Figure 1 shows a flow network on which an \((s, t)\) flow has been computed. The capacity of each edge appears as a label next to the edge, and the numbers in boxes give the amount of flow sent on each edge. (Edges without boxed numbers specifically, the four edges of capacity=3 have no flow being sent on them.)

![Figure 1: Flows and Cuts](image)

(a) What is the value of this flow? Is this a maximum \((s, t)\) flow in this graph?

(b) Find a minimum \((s, t)\) cut in the flow network pictured in Figure 1, and also say what its capacity is.

2. (10 points) K&T Ch7 Ex4. Decide whether you think the following statement is true or false. If it is true, give a short explanation. If it is false, give a counterexample.

(a) Let \(G\) be an arbitrary flow network, with a source \(s\), a sink \(t\), and a positive integer capacity \(c_e\) on every edge \(e\). If \(f\) is a maximum \(s\rightarrow t\) flow in \(G\), then \(f\) saturates every edge out of \(s\) with flow (i.e., for all edges \(e\) out of \(s\), we have \(f(e) = c_e\)).

(b) Let \(G\) be an arbitrary flow network, with a source \(s\), a sink \(t\), and a positive integer capacity \(c_e\) on every edge \(e\); and let \((A, B)\) be a minimum \(s\rightarrow t\) cut with respect to these capacities \(c_e : e \in E\). Now suppose we add 1 to every capacity; then \((A, B)\) is still a minimum \(s\to t\) cut with respect to these new capacities \(\{1 + c_e : e \in E\}\).
3. **(20 points) Updating Flows.** Let $G = (V, E)$ be a unit-capacity flow network with source $s$ and sink $t$. We are also given an integer maximum flow for $G$. Give an algorithm to efficiently update the maximum flow in $G$.

(a) a new edge with unit capacity is added to $E$;
(b) an edge is deleted from $E$.

Note: The algorithm you provide should be faster than recomputing the maximum flow in the updated graph.

4. **(20 points) K&T Ch7 Ex7.** Consider a set of mobile computing clients in a certain town who each need to be connected to one of several possible base stations. We’ll suppose there are $n$ clients, with the position of each client specified by its $(x, y)$ coordinates in the plane. There are also $k$ base stations; the position of each of these is specified by $(x, y)$ coordinates as well.

For each client, we wish to connect it to exactly one of the base stations. Our choice of connections is constrained in the following ways. There is a range parameter $r$ such that a client can only be connected to a base station that is within distance $r$. There is also a load parameter $L$ such that no more than $L$ clients can be connected to any single base station.

Your goal is to design a polynomial-time algorithm for the following problem. Given the positions of a set of clients and a set of base stations, as well as the range and load parameters, decide whether every client can be connected simultaneously to a base station, subject to the range and load conditions in the previous paragraph.

5. **(0 points) How long did it take you to complete this assignment?**