CS 103: Lecture 16 Small Worlds

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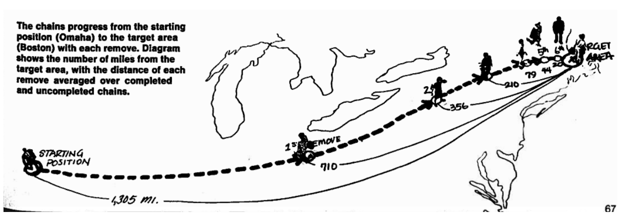
December 7, 2015

Announcements

- HW 6 due Tuesday
- Office hour Sunday 3-4pm in Clapp 202
- Or on Monday by appt.

6 Degrees of Separation

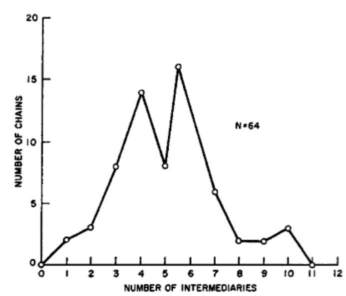
Stanley Milgram, 1960s



- 296 "random" starters in Omaha, NE and Wichita, KS
- "Forward letter to target by sending to someone you know on first name basis with same instructions
- Target = stock-broker in Sharon, MA; known address and occupation

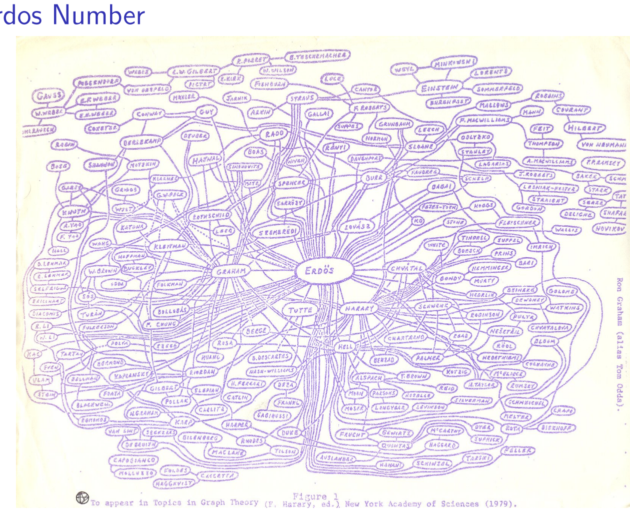
6 Degrees of Separation

Results



Results:
- 64 chains completed
- median path length of complete chains = 6

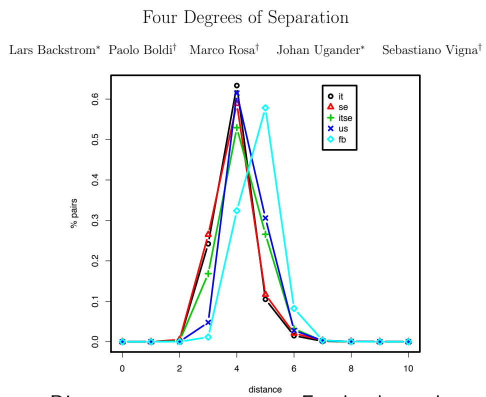
Erdos Number



Other Examples in Pop Culture

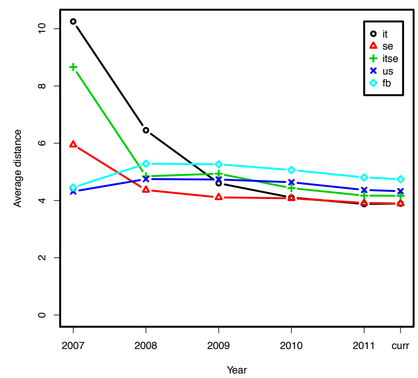
- Kevin Bacon game. "Bacon number"
- Erdos-Bacon number

Four Degrees of Separation



Distance measurements on Facebook graph

The World is Getting Smaller

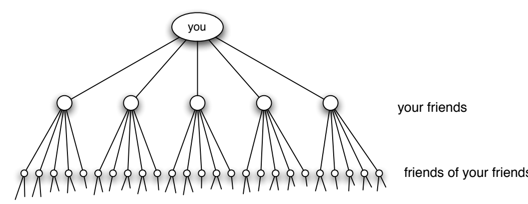


Should real networks have short paths?

Exercise: discuss with a partner. One of you argue why. One of you argue why not.

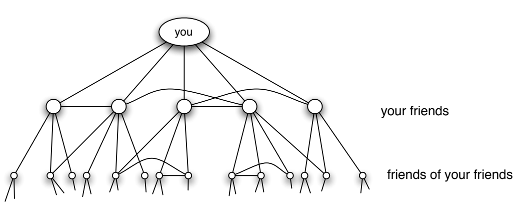
Should real networks have short paths?

Argument for "yes" answer: exponential growth in number of contacts at increasing distance



Should real networks have short paths?

Argument for "no" answer: real graphs exhibit triadic closure



Which one of these forces wins? (exponential growth)

Watts-Strogatz: Small World Networks

Is there a natural model for networks that have triadic closure and short paths?

Watts-Strogatz late 1990s:

- n nodes arranged in a grid (1d, 2d, etc.) that "wraps around"
- Each node has links to
  - all nodes within grid distance d (triadic closure)
  - k random nodes

Example and Demo

Watts-Strogatz: Small World Networks

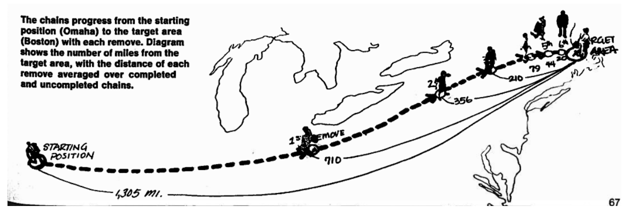
Can show mathematically that Watts-Strogatz networks have:

- many triangles
- short paths (roughly logarithmic in number of nodes)

Board work: what is a short path?

Milgram's Experiment Reconsidered

OK, so short paths exist. But how do people **find** them?



Simple algorithm: pass the message to your neighbor that is closest to the target. Will this work?

Milgran's Experiment Reconsidered

Simple algorithm: pass the message to your neighbor that is closest to the target. Will this work?

This does **not** work for Watts-Strogatz models. Long-range contacts are "too random"

Enter Kleinberg. . .

Kleinberg's Model for Decentralized Search

- Nodes arranged in 2d-grid
- Each node has connections to
  - Grid neighbors
  - One random long-range contact (but not *uniformly* random. . .)

Long-range contact: select a node at distance d with probability proportional to $1/d^r$

Example and demo of

Effect of r

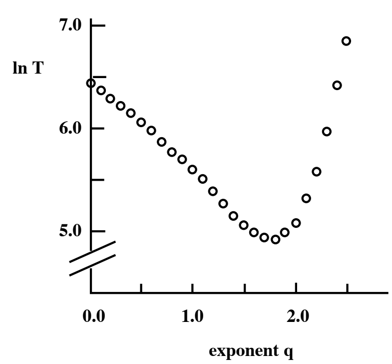
- Small r: links are very long-range. Like Watts-Strogatz
- Big r: links are short-range
- Kleinberg's main result: $r = 2$ is "just right". Links spread over many different distance scales

Result: when $r = 2$, then short paths exist and people can find them. For grids with n nodes, the number of hops to find the target is about $(\log n)^2$

As $n \to \infty$, $r = 2$ is the *only* value that works.

Effect of r

Empirical evaluation on grid with 400M nodes



Demo

To Be Continued

Next time:

- A rough calculation to justify this
- Empirical support for exponent $r = 2$

A Rough Calculation to Justify This

(Informal) If $r = 2$, then a user has roughly equal probability of having a link at any distance scale

Revisit Milgram figure

Rough calculation on board

Empirical Support

Next time: empirical support for the exponent $r = 2$.