Epidemics in Networks

CS 103 Networks
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Announcements

- HW 5 due Tuesday

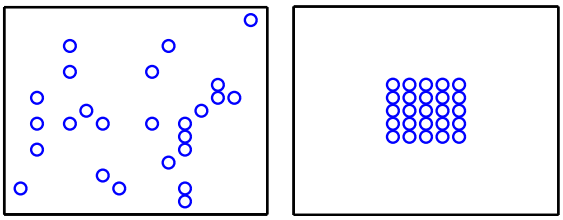
Epidemics

- Beginning of study of network *processes*
  - How do networks affect the world?
- Beautiful models with many connections
  - Spread of disease, rumors
  - Adoption of new behaviors
  - Animal populations in fragmented landscapes

Habitat Loss and Fragmentation

Habitat Loss and Fragmentation

- Which configuration is better?



Epidemics and Rumors

- Branching process
  - Will disease or rumor spread widely?
  - A simple mathematical result

A dichotomy

- Branching factor k, infection probability p.
- Let $R_0=pk$ be the *basic reproductive number*
  - # of new infections created by a single infection
- **Claim:**
  - If $R_0< 1$, the disease dies out quickly
  - If $R_0> 1$, the disease spreads widely

Discuss, make precise, and prove on board

SIR Model

- What is unsatisfying about the branching process model for epidemics?
- SIR model for epidemics on *general graphs*
- Nodes can be in three states:
  - (S) Susceptible: not yet infected
  - (I) Infectious: infected, can infect others
  - (R) Removed: no longer infectious; immune

SIR Model

Example on board

- Initially
  - Some nodes infected (I)
  - All others susceptible (S)
- When node v first becomes infected
  - Pass disease to each susceptible (S) neighbor with probability p
  - After one time step, no longer infectious; becomes removed (R)

Basic Reproductive Number for SIR?

- Will disease die out quickly or spread broadly?
- Q: Is there still a dichotomy with respect to R_0?
  - Suppose each node has k neighbors
  - Infection probability is p
  - Let $R_0=pk$.
  - Q: Do we still have dichotomy: $R_0 < 1$ vs. $R_0 > 1$?
- A: No. *Example.*

Basic Reproductive Number for SIR?

- In general, SIR still exhibits "knife-edge" behavior with respect to p and other parameters
  - Below threshold: die out quickly
  - Above threshold: spread widely
- Much harder to analyze in general graphs

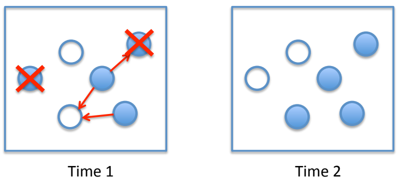
Metapopulations

- Dynamics of spatially disjoint populations

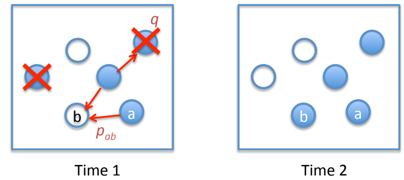
Metapopulation = population of populations

Simple Metapopulation Model

- Patches 1,…,n
- Time steps 1,…,T
- Dynamics:
  - *Local extinction*
  - *Colonization*



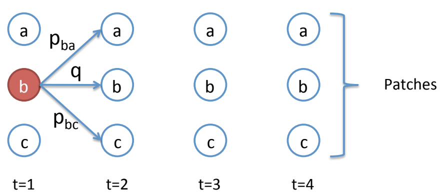
Metapopulation Update Rules



- Occupied patch goes extinct with probability q
- Patch a colonizes patch b with probability p_{ab}
- Patch is occupied in next time step if
  - currently occupied and does not go extinct
  - or colonized by another patch

Metapopulation = SIR

- A metapopulation model is a spreading process



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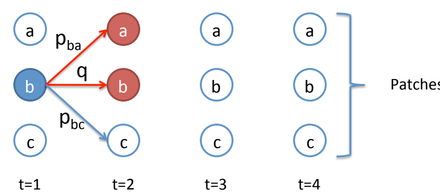

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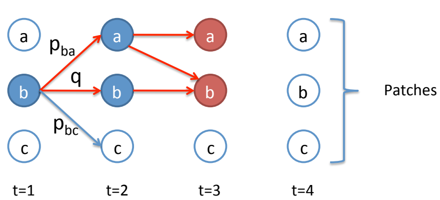

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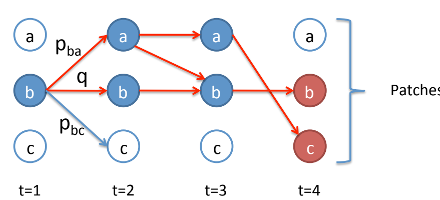
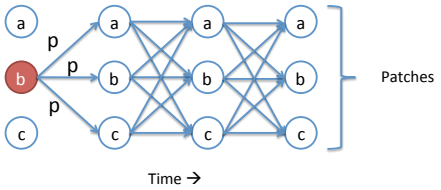


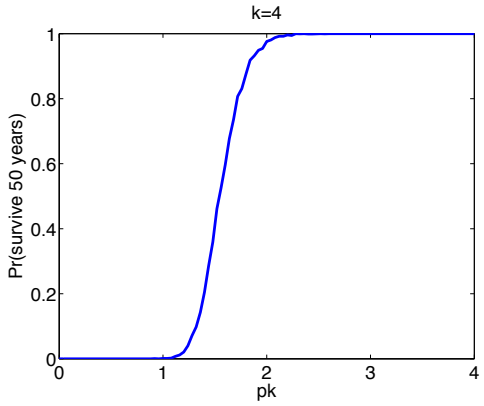
Simple Example

- All probabilities equal to p



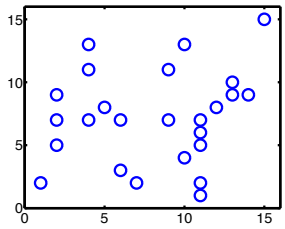
– Will population go extinct?

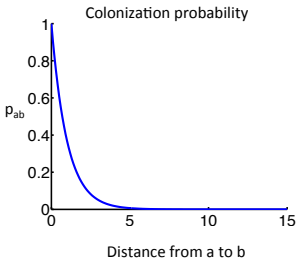
Effect of p



Two More Experiments

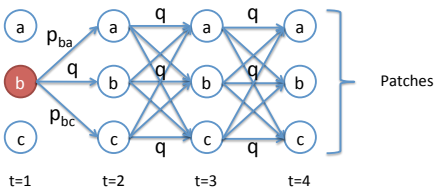
A Test Landscape





Habitat Degradation

- What happens as yearly survival probability q decreases?



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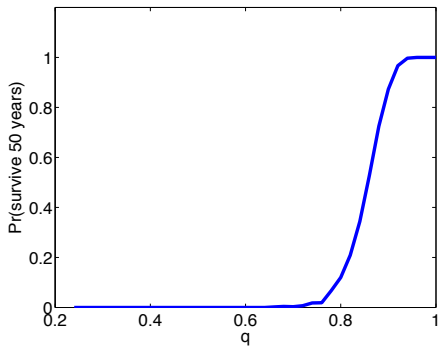
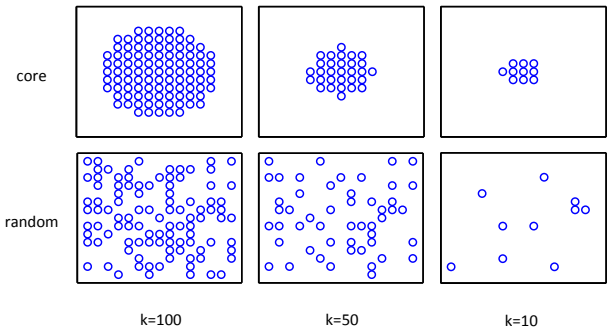


Habitat Fragmentation



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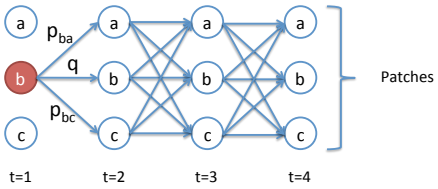

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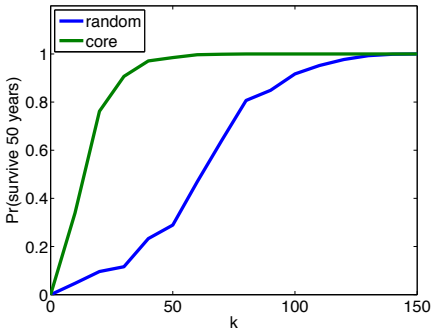

Habitat Fragmentation

- Which configuration is better?