CS 103: Lecture 5 More Game Theory

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Announcements

- HW 1 due now
- HW 2 posted tomorrow, due next Thursday
- Blog posts. . . . Tuesday

Plan for today

- More game theory
  - Nash equilibra
  - Mixed strategies

Review

Draw Prisoner's dilemma on board

- What are strategies of player 1?
- List all the outcomes of the game
- What is $P_1(C, NC)$?

Best Response

Definition: strategy S for player 1 is a **best response** (BR) to strategy T of player 2 if no other strategy S' gives higer payoff when paired with T

$$P_1(S, T) \geq P_1(S', T) \text{ for all other strategies } S'$$

Example

Best Responses in Prisoner's Dilemma (PD)

- P1: C is BR to NC
- P1: C is BR to C

C is a BR for P1 *for any strategy* of P2 $\rightarrow$ easy to predict what P1 will do (C)

Same for P2: we should expect (C,C)

Dominant Strategy

Definition: strategy S for Player 1 is a **dominant strategy** (DS) if it is a best response to every strategy by Player 2

Example (PD):

- P1: C is BR to NC
- P1: C is BR to C

C is a dominant strategy for Player 1

What if players do not have dominant strategies?

Example 2 (on board)

Reason on board. Summary:

- P1: A is BR to X
- P1: A is BR to Y
- P1: A is a dominant strategy

- P2: X is BR to A
- P2: Y is BR to B
- P2: no dominant strategy

What will happen? Still easy to predict:

- P1 will play A (DS)
- P2 will play X (BR to A)

Nash Equilibria

What if neither player has a dominant strategy?

Definition: A **Nash equilibrium** is a pair of strategies that are best responses to each other.

- John Nash 1950
- Central notion of game theory
- What we predict as the result of rational play

If the outcome is not a Nash equilibrium, a player can improve payoff by changing her strategy

Examples

We've already seen two examples

- Prisoner's dilemma: (C, C)
- Example 2: (A, X)

Example: coordination game

		Your Partner	
		PowerPoint	*Keynote*
You	*PowerPoint*	$1, 1$	$0, 0$
	Keynote	$0, 0$	$1, 1$

- What are best responses for P1 (you) and P2 (your partner)?
- Are there any dominant strategies?
- Which outcomes are Nash equilibria?

Examples

Draw two more examples on board

Exercise:

- What real-world situation does this game model?
- What are best responses for P1 and P2?
- Are there any dominant stratgies?
- What are Nash equilibria?

Nash Equilibrium

Is this the right concept to predict the outcomes of a game?

Does *every* game have a Nash equilibrium?

If not, what will happen?

Example on board: rock-paper-scissors

Some games have no Nash equilibria. In these situtations, players play *mixed stratgies* (choose strategy randomly)

Example: Penalty Kicks

Draw game on board

Mixed strategy for P1 (kicker):

- Kick L with probability p
- Kick R with probability $1-p$

Mixed strategy for P2 (goalie):

- Defend L with probability q
- Defend R with probability $1-q$

Payoffs?

How do we evaluate payoffs under mixed strategies?

Suppose goalie's mixed strategy is q, What are kicker's payoffs for kicking L / R? (work out on board)

- Kick L: $\frac{1}{2} \cdot q + \frac{3}{4} \cdot (1-q)$
- Kick R: $1 \cdot q + \frac{1}{2} \cdot (1-q)$

Goalie's payoffs if kicker's mixed strategy is p:

- Defend L: $\frac{1}{2} \cdot p + 0 \cdot (1-p)$
- Defend R: $\frac{1}{4} \cdot p + \frac{1}{2} \cdot (1-p)$

Mixed Strategy Nash Equilibrium

Goalie should choose q so kicker get's equal payoff from each strategy. Why?

Work out on board

Result: $q = 1/3$, $p = 2/3$

Note: kicker chooses less powerful strategy most of the time. Why?

Mixed Strategy Nash Equilibrium

Famous result by John Nash: there is *always* a mixed strategy Nash equilibrium. (Nobel prize 1994)

Empirical Analysis (Palacio-Huerta, 2002)

		Goalie	
		L	R
Kicker	L	$0.58, -0.58$	$0.95, -0.95$
	R	$0.93, -0.93$	$0.70, -0.70$

- Payoffs based on success rates on 1400 penalty kicks
- With these payoffs, we predict
  - Kick L with probability $p = 0.39$
  - Defend L with probability $p = 0.42$
- Actual frequencies:
  - Kick L with probability $p = 0.40$
  - Defend L with probability $p = 0.42$