CS 103: Lecture 3 Positive and Negative Relationships and Structural Balance

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September 17, 2015

Announcements

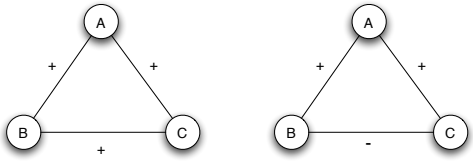
- HW 1 posted today, due next Thursday in class
  - Start early!
- TA Office Hours
  - Areeba Tuesday 7–8pm, CS lounge (Clapp 222A)
  - Tiffany Wednesday 8–9pm, CS lounge (Clapp 222A)
- Blog posts announced next week

Plan for today

- Review / finish weak ties...
- Structural Balance

Positive and Negative Relationships

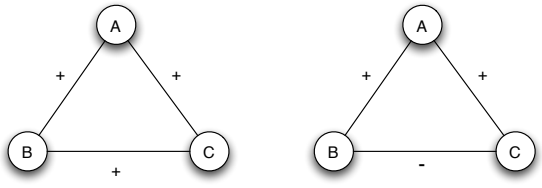
So far edges in a network have been implicitly positive. But you can have enemies too...



Theory of **Structural Balance**

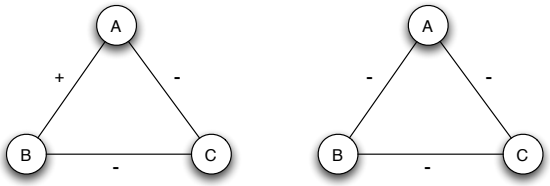
- What happens if network has positive/negative edges?
- What local configurations do we expect to see?
- How does this impact the *global* structure of the network?

Examples



- Left = balanced. Mutual friends
- Right = not balanced. A is stressed out

Examples

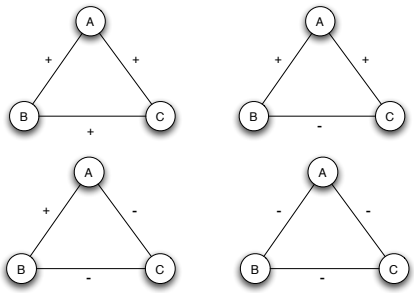


- Left = balanced. Two friends with common enemy
- Right = not balanced. Enemy of my enemy is my friend (two can align against third)

Balanced Triangles

Definition:

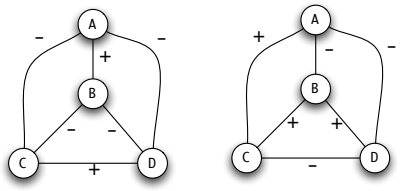
- A triangle with 1 or 3 '+' edges is **balanced** (left column)
- A triangle with 0 or 2 '+' edges is **not balanced** (right column)



How can we extend this to bigger graphs?

Let's focus on *complete* graphs (edges between every pair of nodes)

- Students in a classroom
- Diplomatic relations between countries



balanced not balanced

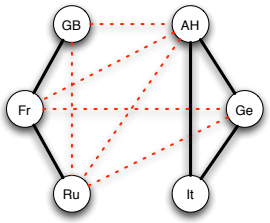
Definition: a complete graph is balanced if every one of its triangles is balanced.

Examples

For each of these graphs, answer if it is balanced or not

Examples on board

What is the minimum number of labels to change to make this graph balanced?



(e) *Entente Cordiale 1904*

Discussion

Definition: a complete graph is balanced if every one of its triangles is balanced.

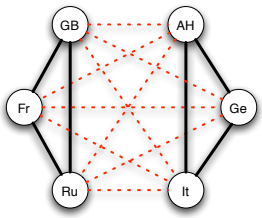
This is an extreme definition. How can we relax it?

- Not all pairs of nodes are friends (any graph)
- *Most* of the triangles are balanced

Balanced Network: Intuition

What does a balanced network "look like"? Can we say more than "all triangles are balanced"? Discuss.

We already saw one way for a graph to be balanced.

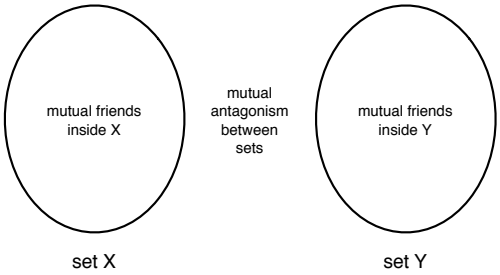


(f) *British Russian Alliance 1907*

Does this give you any hints?

Balanced Network: Intuition

Clearly, Any graph that looks like this is balanced:



(Two sets of mutual friends with antagonim between them)

Structural Balance Theorem (Harary 1953)

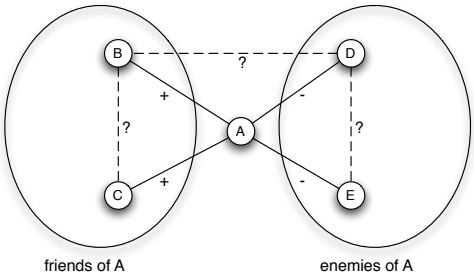
Theorem: all balanced graphs look like the picture we just saw.

- Two sets X and Y
- All pairs within X, Y are friends
- Everyone in X enemies with everyone in Y

Proof on board

Significance: a purely "local" concept (balanced triangles) determines the *global* structure

Review of Proof Idea (for notes)



Generalizations

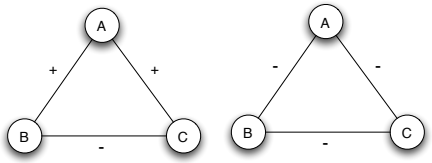
What do you like least about this?

Generalizations:

- Relax definition of balance (board work)
- Not all edges present (main ideas)
- Most, but not all triangles balanced (reading)

Relax the Definition of Balance

Here are the two types of triangles we declared imbalanced:

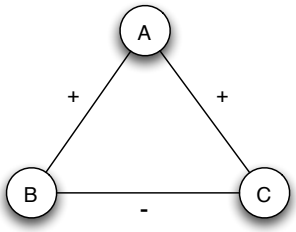


Are these really the same?

Intuition: three negative edges (right) is much more stable than two positive, one negative (left)

Weakly Balanced Networks

Definition: A network is **weakly balanced** if it has no triangles like this:

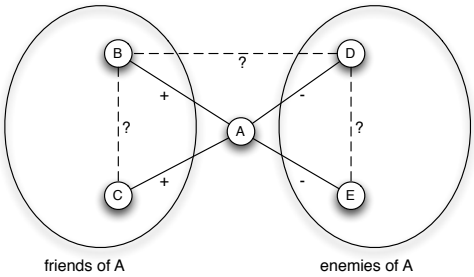


- Two positive edges, one negative

Question: what does the global structure of a weakly balanced network look like?

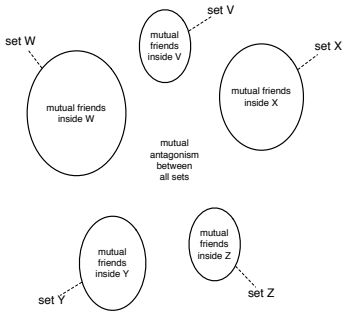
Weakly Balanced Networks

Remember our proof. What changes in this picture?



Weak Structural Balance

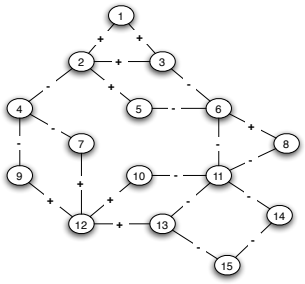
Theorem: a complete graph that is weakly balanced looks like this:



Proof sketch on board

Other Networks

What if the graph is not complete? (strong balance)



Is it balanced? (possible to divide into two sets of mutual friends with antagonism between them)

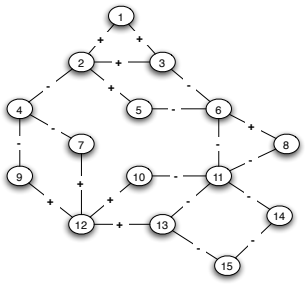
Examples and Odd Cycles

Board work: intuition, odd cycles

Definition: an **odd cycle** is a cycle with an odd number of negative edges

Structural Balance in General Networks

Theorem: a general network is structurally balanced if any only if it has no odd cycles



This example is **not** balanced. Can you see why?