The Really Really Big Urn

Solution

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Under the usual interpretation of "what is left after all the removals," the urn is empty.

Choose any particular pebble, say pebble 137. Eventually it is put in the urn. Later, at step $2 \cdot 137 = 274$, pebble 137 is taken out. It never returns.

The same is true of *every* numbered pebble. For each pebble k, there is a specific finite step, $2k$, at which it is removed. Consequently no pebble belongs to the limiting collection.

But the number of pebbles keeps growing!

Yes. After n complete rounds of adding ten and removing one, the urn contains $9n$ pebbles. That count tends to infinity.

These are two different limiting questions:

> How large are the finite-stage collections? Their sizes grow without bound.
>
> Which fixed pebbles remain in the limiting collection? None.

Taking a limit of collections one pebble at a time and then counting need not give the same result as counting first and taking a limit of the numbers. Here the pebbles occupying the urn keep changing: every fixed one eventually leaves, while newer, higher-numbered pebbles take its place.

There is no final ordinary integer-numbered step. Calling the limiting urn empty means specifically that a pebble belongs to it only if its membership eventually settles at "present." Every pebble's membership instead settles at "absent." Without a meaning for "after infinitely many steps," the question would not define an additional physical state.

History and sources: companion history PDF.