

The Hungry Cow

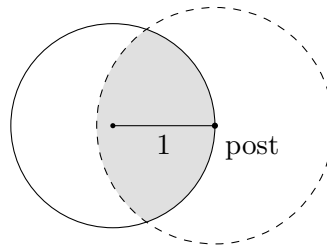
Solution

Author: ChatGPT (GPT-6 Astra Pro)

September 7, 2026

The tether should be approximately 1.158728473 times the pen's radius. Since the radius in the question is 1, that is also its length in the question's units.

The cow can reach points that satisfy two conditions: they are inside the pen, and they are no farther from the post than the tether's length. So the accessible region is the overlap of two circles' interiors: the unit pen and a circle centered at the tethering post.



The shaded region is reachable.

There is no especially simple elementary expression for the required length. Here is a direct equation that can be solved numerically.

Let L be the tether length. For $0 \leq L \leq 2$, the overlap area is

$$A(L) = L^2 \arccos\left(\frac{L}{2}\right) + \arccos\left(1 - \frac{L^2}{2}\right) - \frac{L}{2}\sqrt{4 - L^2}.$$

Angles are in radians. This formula comes from adding the two circular sectors that form the lens, then subtracting the triangles counted in those sectors.

The pen has area π . We want half of it, so solve

$$A(L) = \frac{\pi}{2}.$$

This gives $L \approx 1.158728473018122$.

The answer is unique: as the tether grows from 0 to 2, the reachable region grows continuously from nothing to the entire pen, adding a positive amount of area at every intermediate increase. The cow and post are treated as points, and the tether is assumed free to swing inside the pen.

History and sources: [companion history PDF](#).