

The Hard Problem

Solution

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Agree on one representative hat arrangement for each class of arrangements that differ in only finitely many places. Everyone guesses the color assigned by the representative.

Here is what that means, step by step.

Number the mathematicians $1, 2, 3, \dots$. Write a complete hat arrangement as an infinite sequence such as

black, white, white, black, $\dots$

Call two arrangements *almost the same* if they disagree at only finitely many positions. For example, changing the hats of people 2, 8, and 100 gives an almost-the-same arrangement. Changing every other hat does not.

This divides all arrangements into classes. Within a class, any two arrangements disagree only finitely often. Before the hats are placed, use the axiom of choice to choose one representative arrangement from each class. Everyone agrees on the same representatives.

How can someone find the class without seeing their own hat?

Fill in the missing hat color arbitrarily. Either possible completion differs from the actual arrangement at at most one position. So both completions belong to the *same class as the actual arrangement*.

Every mathematician can therefore identify that same class, even without knowing their own hat. Each guesses the color that the chosen representative assigns to their own position.

The actual arrangement and its representative are in the same class, so they differ at only finitely many positions. Those positions are exactly the people who guess incorrectly. Everyone else is right.

What the axiom of choice is doing

It supplies a simultaneous choice of one representative from every class. This is a nonconstructive mathematical strategy, not an algorithm that ordinary people or a finite computer can execute by examining infinitely many hats.

There is no fixed numerical bound on the number of errors: the guarantee is that the number is finite for *each* arrangement. No probability distribution over hats is needed. In particular, the argument does not require the continuum hypothesis, or assigning an infinite sum of hat colors an “even” or “odd” parity.

History and sources: [companion history PDF](#).