

Fox and the Foxholes

Solution

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For $N \geq 3$, the fewest days needed to guarantee finding the fox is $2N - 4$.

Number the holes $1, 2, \dots, N$. Check holes in this order, one per day:

$$2, 3, \dots, N - 1, \quad N - 1, N - 2, \dots, 2.$$

Notice that you check hole $N - 1$ on two consecutive days. For five holes, the complete plan is 2, 3, 4, 4, 3, 2.

Why the plan works

Every night the fox moves one hole, so its hole number switches between even and odd. On the outward sweep, your checked hole number also switches between even and odd each day.

If the fox starts in an even-numbered hole on day 1, it always has the same parity as the hole checked during that sweep. Unless found immediately, it starts ahead of your search at hole 2. It cannot slip past you: changing from ahead to behind would first require being at the checked hole on a day when you check it. By the time you reach hole $N - 1$, there is no same-parity hole farther ahead. You must have found it.

If the outward sweep fails, the fox started with the other parity. Checking $N - 1$ again on the next day changes which parity you are matching, since the fox moves but your check does not. The inward sweep catches this remaining case by the same argument, working from right to left.

Why fewer days cannot guarantee success

Every interior hole—holes 2 through $N - 1$ —must be checked on both an even-numbered day and an odd-numbered day.

Suppose, for example, that some interior hole is never checked on an even day. A fox could occupy that hole on every even day. On each odd day, it could choose whichever of that hole's two neighbors is not being checked. It can always move back to the interior hole the next night. This produces a valid route that avoids the entire search.

The same argument applies with even and odd exchanged. Thus each of the $N - 2$ interior holes needs at least two checks, giving the lower bound $2(N - 2) = 2N - 4$. Our plan reaches that bound.

Small cases. With two holes, check the same hole on two consecutive days. One day cannot guarantee success. With only one possible hole, a single check suffices, interpreting that degenerate case as a fox that stays there.

History and sources: [companion history PDF](#).