

The Four Quarters Problem

Solution

Author: ChatGPT (GPT-6 Astra Pro)

September 7, 2026

Yes. First ask whether all four coins are heads. If not, use the following 15 moves, stopping as soon as the answer is yes.

Use these four kinds of move, always referring to the tray's *current* position:

<i>A</i>	Flip all four coins.
<i>D</i>	Flip a diagonal pair.
<i>S</i>	Flip an adjacent pair.
<i>O</i>	Flip one coin.

For example, consistently choose the upper-left/lower-right diagonal for *D*, the upper two corners for *S*, and the upper-left corner for *O*. The tray may rotate between every move; you need not track the physical coins.

A, D, A, S, A, D, A, O, A, D, A, S, A, D, A

Ask whether all coins are heads after *each* move.

Why this works

First consider the easier goal of making all four coins *match*, either all heads or all tails. The seven-move sequence

D, S, D, O, D, S, D

accomplishes that, regardless of the rotations.

If exactly two coins are heads, those heads are either diagonal or adjacent. A *D* move makes diagonal heads all match. If the heads are adjacent, *D* leaves them adjacent. An *S* move on adjacent heads either makes all coins match or leaves diagonal heads; the following *D* then makes them match. Thus *D, S, D* handles any two-head position.

If the number of heads is odd, the first three moves keep it odd, because they each flip two coins. The *O* move makes the number even. The coins now either all match, or have exactly two heads, which the final *D, S, D* handles.

To turn “all match” into “all heads,” insert an *A* move before the sequence and after each of its seven moves. Whenever a matching position is all tails, the next *A* makes it all heads. Complementing all four coins does not change whether a two-head pair is adjacent or diagonal, or whether the head count is odd. So the preceding argument still works.

Fifteen is also optimal in the worst case. Even if the tray never rotated, each check can succeed for only one of the 16 starting arrangements along a sequence of failed checks. The initial check plus *k* moves give at most *k* + 1 such opportunities. Covering all 16 arrangements requires at least 15 moves.

History and sources: [companion history PDF](#).