

Coin Flip Contest

Solution

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The probability is exactly $1/2$.

Imagine writing down every result of all the flips. Make a partner for this complete result by changing every head to a tail and every tail to a head. Because all flips are independent and fair, the two complete results are equally likely.

Suppose Alice originally gets a heads and Bob gets b heads. In the partner result, Alice gets $851 - a$ heads and Bob gets $850 - b$.

Alice wins in the original result exactly when $a > b$. She wins in the partner result exactly when

$$851 - a > 850 - b,$$

which is equivalent, for integer counts, to $a \leq b$.

So Alice wins in exactly one member of every pair. Half of the equally likely complete results are wins for Alice.

This argument includes ties in the original contest: a tied result is paired with one in which Alice wins by one head. It does not assume that Alice and Bob are equally likely to win; Bob can also tie Alice, so his probability of *strictly* winning is less than $1/2$.

The numbers 850 and 851 are not special. The same proof works whenever Alice makes exactly one more independent fair flip than Bob.

History and sources: [companion history PDF](#).