The Ant on the Rubber Band

Solution

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With the far end moving away at 10 meters per second, the ant eventually reaches it. With the band doubling in length every second, the ant does not.

The ant walks at 0.1 meters per second *relative to the rubber beneath it.* Stretching carries it along too. The useful quantity is the *fraction* of the band it has covered, not just its distance from the wall. Uniform stretching preserves that fraction; walking increases it.

Why the constant-speed stretching case eventually ends

During a period in which the band grows from length L to length $2L$, it takes $L/10$ seconds to grow that much. Throughout that period, the band is at most $2L$ long. Walking therefore increases the ant's fraction at a rate of at least $0.1/(2L)$ per second.

During that one doubling period, its fraction increases by at least

$$\frac{0.1}{2L} \cdot \frac{L}{10} = \frac{1}{200}.$$

The same lower bound applies to *every* doubling period. After at most 200 such periods, the ant must have covered the whole band. Each of these periods takes a finite, though increasingly enormous, time. So the answer is yes without needing calculus.

The exact time

At time t seconds, the band's length is $1 + 10t$ meters. If $f(t)$ is the fraction covered, then

$$f(t) = \int_0^t \frac{0.1}{1 + 10s}\, ds = 0.01 \ln(1 + 10t).$$

Set $f(t) = 1$ to obtain

$$\boxed{t = \frac{e^{100} - 1}{10} \approx 2.6881 \times 10^{42} \text{ seconds.}}$$

This is a mathematical conclusion under unlimited stretching, not a physically realistic prediction.

Doubling at the end of each second

For the other version, suppose the band is initially 1 meter long and doubles instantaneously and uniformly at the end of every second. The ant covers these fractions in successive seconds:

$$0.1, \quad 0.05, \quad 0.025, \quad \ldots$$

Their total is only 0.2. The ant never reaches the end.

If "doubling" instead means continuous exponential growth, length 2^t, the limiting fraction is $0.1/\ln 2 \approx 0.14427$, also less than 1. The answer is still no, but the two stretching rules give different limiting fractions.

History and sources: companion history PDF.