# 8 Coins

Solution

Author: ChatGPT (GPT-6 Astra Pro) September 7, 2026

**Use three even/odd counts to encode the chosen coin's position. One flip can change exactly the counts you need.**

Label the positions $0, 1, \ldots, 7$ from left to right. Agree on these three groups:

| Group value | Positions in the group |
|---|---|
| 1 | $1, 3, 5, 7$ |
| 2 | $2, 3, 6, 7$ |
| 4 | $4, 5, 6, 7$ |

For any arrangement, count the heads in each group. Add the values of the groups whose counts are odd. Call the result the arrangement's *code*. It is a number from 0 to 7.

Your friend's rule is simple: read that code and name the coin with that label.

## Which coin should you flip?

Write the current code and the target label using three binary digits. For each digit, put 1 if the two digits differ and 0 if they agree. The resulting three-digit number is the label of the coin you should flip. This operation is called *exclusive OR*, or XOR.

Why? A coin belongs to group 1, 2, or 4 exactly when its label's corresponding binary digit is 1. Flipping it changes odd to even, or even to odd, in precisely those groups. Choosing the differing digits therefore turns the old code into the target code.

**Example.** If only coins 1, 2, and 4 show heads, the code is 7, or 111 in binary. To indicate target 5, or 101, the digits differ only in the middle position. Flip coin 2, whose label is 010. Now the heads are 1 and 4, and the code is 5.

**What if the code is already right?** Flip coin 0. It belongs to none of the groups, so it changes no count. This is why the strategy can require *exactly* one flip, rather than allowing you to skip a turn.

The target is chosen independently of your flip. Your friend discovers that target, not which coin you flipped. Asking only for the flipped coin would allow the trivial agreement "always flip coin 0."

History and sources: companion history PDF.