The 7 Hats Problem

Solution

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Give each person a different possible remainder, and have them guess as though that remainder were the correct one.

Before the hats are placed, number the seven people $0, 1, \ldots, 6$. Also number the seven colors $0, 1, \ldots, 6$.

After the hats are placed, each person adds the numbers of the six hats they can see. Person 0 guesses the color that would make the total a multiple of 7. Person 1 guesses the color that would make the total leave remainder 1 when divided by 7. Continue in this way through person 6.

There is always exactly one color that makes a particular remainder. For example, if person 3 sees hat numbers adding to 16, they guess color 1: the resulting total would be 17, which leaves remainder 3 after division by 7.

Why must someone be right? The *actual* total of all seven hat numbers has exactly one remainder after division by 7. The person assigned that remainder has made the right assumption and therefore guesses their own hat correctly.

In fact, exactly one person is correct. Everyone else aims for a different remainder and must guess a different color from their actual hat.

The same method works for n people and n colors: use remainders 0 through $n - 1$ after division by n.

History and sources: companion history PDF.