Concentration Inequalities for Conditional Value at Risk
Errata

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Error 1

The equation labeled (2) on page 3 of our paper (Thomas and Learned-Miller, 2019) says:

$$C(Y) = \frac{1}{\alpha} \int_{1-\alpha}^{1} \mathrm{VaR}_\gamma(Y) d\gamma.$$

The $\mathrm{VaR}_\gamma(Y)$ term should be $\mathrm{VaR}_{1-\gamma}(Y)$. That is, the equation labeled (2) should be:

$$C(Y) = \frac{1}{\alpha} \int_{1-\alpha}^{1} \mathrm{VaR}_{1-\gamma}(Y) d\gamma.$$

An equivalent correct equation would be:

$$C(Y) = \frac{1}{\alpha} \int_{0}^{\alpha} \mathrm{VaR}_\gamma(Y) d\gamma.$$

This error should be clear from Figure 3 and noticing that VaR_α was defined in terms of the upper-tail—the incorrect equation assued that VaR_α was defined in terms of the lower-tail. The subsequent expressions all remain correct to the best of our knowledge.

Error 2

In Section 5, we incorrectly assert that our high-confidence upper bound (Theorem 3) does not depend on b, the deterministic upper bound on the random variable. This should say that our high-confidence upper bound does not depend on a, the deterministic lower bound on the random variable. Specifically, the published paper says (bold added to highlight the error):

> We now turn to comparing our upper bound to Brown's, i.e., Theorem 3 to Theorem 1. In this case, Brown's inequality requires X to be bounded both above and below, while our inequality only requires X to be bounded above with probability one. Another difference between our and Brown's upper bounds, which was not present when considering lower bounds, is that Brown's requires X to be a continuous random variable, while ours does not. This means that, again, our

> inequality will be applicable when Brown's is not. However, just as with the lower bounds, our bound only holds for $\delta \in (0, 0.5]$, while Brown's holds for $\delta \in (0, 1]$.
>
> Unlike with the lower bounds, further comparison is more challenging. Neither bound is a strict improvement on the other in the settings where both are applicable. **Our inequality has no dependence on the upper bound, and so for random variables with large upper bounds that are rarely realized, our inequality tends to perform better.** However, our confidence interval scales with $1/\alpha$, while Brown's scales with $1/\sqrt{\alpha}$. Since $\alpha < 1$, this means that Brown's inequality has a better dependence on α.

In the quote above, the bold sentence should be replaced with:

> Our inequality has no dependence on the lower bound, and so for random variable with small lower bounds that are rarely realized, our inequality tends to perform better.

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References

P. Thomas and E. Learned-Miller. Concentration inequalities for conditional value at risk. In *International Conference on Machine Learning*, pages 6225–6233, 2019.