The Hard Problem

History and provenance

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An infinite hat problem

Hardin and Taylor's *An Introduction to Infinite Hat Problems* (2008) discusses the simultaneous, all-visible-except-one's-own puzzle and its connection to the axiom of choice [1]. Their historical account credits Yuval Gabay and Michael O'Connor with formulating a collection of infinite hat problems while graduate students at Cornell in spring 2004, including this type of finite-error guarantee.

The solution idea

Grouping sequences by agreement outside a finite set, then choosing one representative per group, is an established construction in this literature. The accompanying solution explains that construction anew; it does not claim to invent it.

For the countably infinite version on this page, Hardin and Taylor explicitly identify an earlier antecedent: a 1964 result of Fred Galvin, published as Problem 5348 in *The American Mathematical Monthly* 72 (1965), page 1136. They also cite B. L. D. Thorp's 1967 solution [1, p. 21 and references]. This is an antecedent of the mathematical result, not a claim that Galvin introduced the later hat-puzzle story.

The argument is nonconstructive. It uses the axiom of choice but does not require assuming the continuum hypothesis or naming the cardinality of the continuum as a particular successor cardinal.

Sources

[1] Christopher S. Hardin and Alan D. Taylor, An Introduction to Infinite Hat Problems, The Mathematical Intelligencer 30(4) (2008), 20–25.
https://www.cs.umd.edu/~gasarch/TOPICS/hats/infinite-hats-and-ac.pdf

Sources checked September 7, 2026. A documented appearance is not necessarily the earliest appearance.