

687 2017-10-03

5-minute quiz.

Today: Solving the Bellman Optimality Equ more efficiently
(will use Dynamic Programming)

Policy Evaluation

- Given a policy π , find v^π
- Assume $P+R$ are known

Method: Solve Bellman Equ

$$v^\pi(s) = \sum_a \pi(s,a) \sum_{s'} P(s,a,s') [R(s,a,s') + \gamma v^\pi(s')]$$

Dynamic Programming

Sequence of value funcs that approximate v^π :

$$\hat{v}_0^\pi, \hat{v}_1^\pi, \hat{v}_2^\pi, \hat{v}_3^\pi, \dots$$



Choose arbitrarily, e.g.

0 in every state. (Must be 0 for terminal states.)

$$\text{Do: } \hat{v}_{k+1}^\pi(s) \leftarrow \sum_a \pi(s,a) \sum_{s'} P(s,a,s') [R(s,a,s') + \gamma \hat{v}_k^\pi(s')]$$

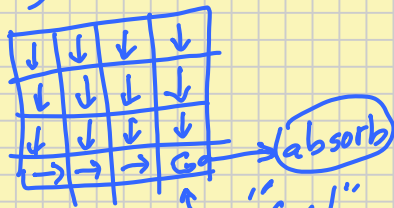
Properties

① $\hat{v}_k^\pi = v^\pi$ is a fixed point

② \hat{v}_k^π converges to v^π as $k \rightarrow \infty$ for finite MDPs with bounded rewards

③ one pass over the state space a full backup.
A single state update is a backup.

Try it out!



- To speed this up, can do an in-place state update.
- Can update in any order.
- Can do updates asynchronously
 ↳ Can update one state multiple times before updating some other state.

- $R = -1$ always
- $\gamma = 1$
- Actions succeed (except at a wall)
- π as in the arrows above

$\hat{V}_0^\pi =$	<table border="1"><tr><td>0</td><td>0</td><td>0</td><td>0</td></tr><tr><td>0</td><td>0</td><td>0</td><td>0</td></tr><tr><td>0</td><td>0</td><td>0</td><td>0</td></tr><tr><td>0</td><td>0</td><td>0</td><td>0</td></tr></table>	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	$\hat{V}_3^\pi =$	<table border="1"><tr><td>-3</td><td>-3</td><td>-3</td><td>-3</td></tr><tr><td>-3</td><td>-3</td><td>-3</td><td>-2</td></tr><tr><td>-3</td><td>-3</td><td>-2</td><td>-1</td></tr><tr><td>-3</td><td>-2</td><td>-1</td><td>0</td></tr></table>	-3	-3	-3	-3	-3	-3	-3	-2	-3	-3	-2	-1	-3	-2	-1	0
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$\hat{V}_6^\pi =$

-6	-5	-4	-3
-5	-4	-3	-2
-4	-3	-2	-1
-3	-2	-1	0

Information flows backwards from the "goal", hence the term backup.

⇓
Guaranteed to converge to V^π if no state is starved for updates.

Can we do the same thing for q ? Yes.

Policy Improvement

$$Q\text{-update: } \hat{q}_{sk\pi}^{\pi}(s, a) = \sum_{s'} P(s, a, s') \left[R(s, a, s') + \gamma \sum_{a'} \pi(s', a') \hat{q}_{sk\pi}^{\pi}(s', a') \right]$$

If we have estimated \hat{q}^{π} , how can we improve π ? What if we are just greedy?

Policy Improvement Thm:

Let $\pi + \pi'$ be deterministic

policies s.t. $\forall s: \hat{q}^{\pi}(s, \pi'(s)) \geq v^{\pi}(s)$.

$\rightarrow = \hat{q}^{\pi}(s, \pi(s))$ for a deterministic π .

Then $\pi' \geq \pi$. (Recall: $\pi' \geq \pi \triangleq \forall s. v^{\pi'}(s) \geq v^{\pi}(s)$)