Practice Midterm 2

1. Given the recursion \( a_n = a_{n-1} + 2a_{n-2} \) with \( a_1 = 6 \) and \( a_2 = 12 \), use induction to prove that \( a_n = 3\cdot 2^n \) for all positive integers \( n \).

2. Use induction to prove that \( \sum_{i=1}^{n} (n-i) = \frac{n(n-1)}{2} \) for all positive integers \( n \).

3. Use the Division Theorem to show that \( n^2 + n + 3 \) is divisible by 3 if and only if \( n \mod 3 = 0 \).

4. Consider the identity \((A \cap B)' = A' \cup B'\)
   
   a) Verify the identity using Venn diagrams
   
   a) b) Prove the identity using an element-wise proof. You may use the fact that \( A \cap B \subseteq A \) for any sets \( A \) and \( B \).

5. More functions and sets: let \( A = \{a,\{s,t\},3\} \), \( B = \{1,2,3,4\} \), \( C = \{a,b,c\} \) and answer the following questions
   
   a) Compute the power set \( \mathcal{P}(A) \)
   
   b) For the function \( f : B \to C \) given in set form by \( f = \{(1,c),(2,b),(3,b),(4,a)\} \) and the function \( g : C \to B \) given by \( \{(a,4),(b,1),(c,2)\} \), give a set representation of the function \( g \circ f \).
   
   c) For the function \( f \) defined in part (b), either give the inverse function \( f^{-1} \) in set form or explain why \( f^{-1} \) does not exist.