Practice Midterm

1. Define the following terms
   a) A **proposition**
   b) A **recurrence relation** defining a sequence of numbers
   c) The **converse** of an implication
   d) An **irrational** number
   e) Proving a theorem by cases

2. For each predicate, translate or negate as instructed. Let $D$ = the set of all classes, $I(x)$ = “$x$ is required”, $H(x,y)$ = “$x$ is harder than $y$”, and $M(x,y)$ = “$x$ has more students than $y$”.
   a) (To symbols) There is a class that is harder than any other. **[Hint: a class cannot be harder than itself.]**
   b) (To English) $\exists x \in D$, $\forall y \in D$, $H(x,y) \rightarrow M(y,x) \land I(y)$
   c) Negate part (b) in symbols
   d) Negate part (b) in English

3. Persons A, B, and C approach you. A says “At least one of us is lying”. B says “A and C are both lying”, and C says “If B is lying, then A is telling the truth”. Who (if anybody) is telling the truth?

4. Consider the sequence given by $a_1 = 1$ and $a_n = a_{n-1} + 9$
   a) Write a recursive formula for the $2k-1$st term.
   b) Write a recursive formula for $a_{2^j+1}$
   c) Write a closed-form formula for $a_n$. **[Hint: Recall the technique of comparing to a simpler sequence]**

5. Prove the following theorem: if $n$ is an integer, then $\frac{n(n+1)}{2}$ is an integer. **[Hint: Consider the two cases where $n$ is even and $n$ is odd.]**