1. Define the following terms
   a. Relatively prime integers
   b. $\mathbb{Z}$
   c. $C(n,r)$
   d. $n(A)$, where $A$ is a given set
   e. Contrapositive of an implication
   f. Sample space

2. Use induction to prove that \[ \sum_{i=1}^{n} \frac{1}{(2i-1)(2i+1)} = \frac{n}{2n+1} \] for all positive integers $n$.

3. Prove that if $2^{m-1} \mod{11} = 7$, then $2^{m} \mod{11} = 3$. [Hint: $2^{m} = 2 \cdot 2^{m-1}$.]

4. Functions and sets.
   a) Express the set identity $A \cup (A' \cap B) \subseteq A \cup B$ as a logical implication using the propositions $p = (x \in A)$ and $q = (x \in B)$. Do the same for the identity $A \cup B \subseteq A \cup (A' \cap B)$.

   b) Prove both of the above logical implications using truth tables (i.e., show that each is a tautology). Have you just proved that $A \cup (A' \cap B) = A \cup B$? [Note: You can use a single truth table for both proofs]

   c) Let $c: \mathcal{P}\{x,y,z\} \rightarrow \mathcal{P}\{x,y,z\}$ be the function with rule $c(A) = A'$ and let $n: \mathcal{P}\{x,y,z\} \rightarrow \{0,1,2,3\}$ be the function such that $n(A)$ is the number of elements in $A$. Which composition is defined, $c \circ n$ or $n \circ c$? For the composition that is defined, draw an arrow diagram and explain why or why not the composite function is invertible.

5. There are 16 distinguishable marbles in a box: 5 red, 3 green, and 8 blue.
   a) How many ways can a sample of 4 marbles be selected, without replacement and without regard to order
   b) Of the above samples, how many have all four selected the same color?
   c) How many of the samples from Part (a) have two colors, with two of each color?
   d) If you take a sample of size 3, what is the probability that all colors are distinct?
   e) If you take a sample of size 3, then what is the probability that at least one of the other marbles is blue, given that exactly one of the marbles is green?