Answers to Sample Midterm Questions

1. Claims processing. (This is an example of both a "fork-join" queueing system and a workflow system.)

(a) GSMP specification:

(i) **State space**: $S =$ the set of all $(n_1, n_2, m) \in \{0,1,2,\ldots\}^3$ such that $n_1 \le m$ and $n_2 \le m$.

(ii) **Event set mapping**: For $s = (n_1, n_2, m) \in S$

$e_1 \in E(s)$ for all s

$e_2 \in E(s)$ if and only if $n_1 > 0$

$e_3 \in E(s)$ if and only if $n_2 > 0$

$e_4 \in E(s)$ if and only if $m - n_1 > 0$ and $m - n_2 > 0$

[equivalently: $e_4 \in$ E(s) iff $\min(m - n_1, m - n_2) > 0$; $e_4 \in$ E(s) iff $m - \max(n_1, n_2) > 0$]

(iii) **Speeds**: All speeds $r(s,e)$ are equal to 1.

(iv) **State-transition probabilities**:

$p(s'; s, e_1) = 1$ when $s = (n_1, n_2, m)$ and $s' = (n_1 + 1, n_2 + 1, m + 1)$

$p(s'; s, e_2) = 1$ when $s = (n_1, n_2, m)$ with n_1>0 and $s' = (n_1 - 1, n_2, m)$

$p(s'; s, e_3) = 1$ when $s = (n_1, n_2, m)$ with n_2>0 and $s' = (n_1, n_2 - 1, m)$

$p(s'; s, e_4) = 1$ when $s = (n_1, n_2, m)$ with $\min(m - n_1, m - n_2) > 0$ and $s' = (n_1, n_2, m - 1)$

(b) The table is as follows (new clock readings are in **boldface**.)

n	ζ_n	$X(\zeta_n)$	$C_{n,1}$	$C_{n,2}$	$C_{n,3}$	$C_{n,4}$	e^*
0	0.0	(1,1,1)	**4**	**2**	**3**	--	e_2
1	2.0	(0,1,1)	2	--	1	--	e_3
2	3.0	(0,0,1)	1	--	--	**4**	e_1
3	4.0	(1,1,2)	**5**	**6**	**4**	3	e_4
4	7.0	(1,1,1)	2	3	1	--	e_3

(c) The computations are as follows:

$$\hat{\mu} = \frac{1}{5}\left(1.0 + 3.0 + 7.0 + 5.0 + 4.0\right) = 4.0$$

$$s^2 = \frac{1}{4}\left((1.0 - 4.0)^2 + (3.0 - 4.0)^2 + (7.0 - 4.0)^2 + (5.0 - 4.0)^2 + (4.0 - 4.0)^2\right) = 5.0$$

$$z_p = 1.645 \quad \text{[0.95 quantile for N(0,1)]}$$

$$\varepsilon = 0.1$$

$$n = \frac{z_p^2 s^2}{\varepsilon^2 \hat{\mu}^2} = \frac{(1.645)^2 (5)}{(0.1)^2 (4)^2} \approx 85,$$

so we need at least 85 runs.

2. Random number generation.

(a) The inverse of the cdf is given by $F_X^{-1}(u) = (u/\beta)^{1/\alpha}$ for $0 \le u \le 1$. Using inversion, generate U and return $X = (U/\beta)^{1/\alpha}$.

(b) The pdf is $f_X(x) = dF_X(x)/dx = \alpha\beta x^{\alpha-1}$ so the expected value of the distribution is $\int_0^{\beta^{-1/\alpha}} x\, f_X(x)\, dx = \int_0^{\beta^{-1/\alpha}} \alpha\beta x^{\alpha}\, dx = \frac{\alpha}{\alpha+1}\beta^{-1/\alpha}$. For $\alpha = 2$ and $\beta = 4$, the expected value equals 1/3. The given value is less than half of this amount, so the generator is untrustworthy.