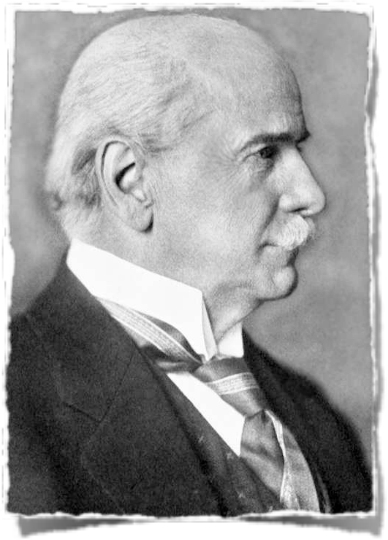

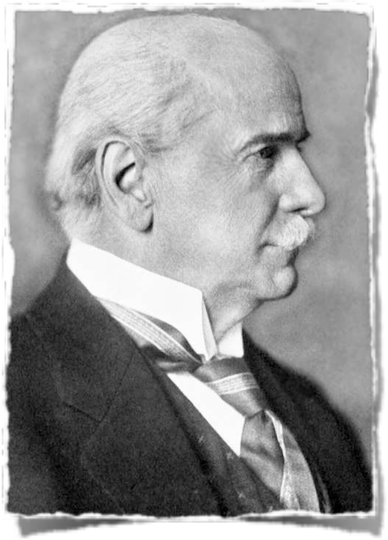

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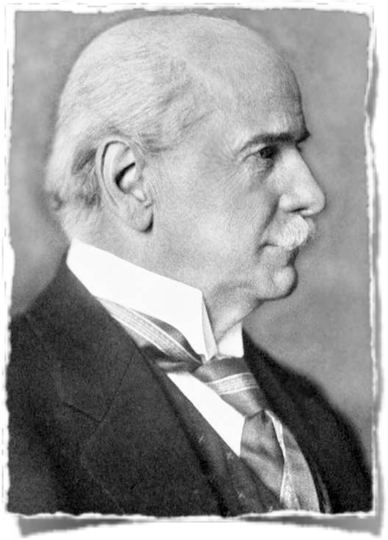

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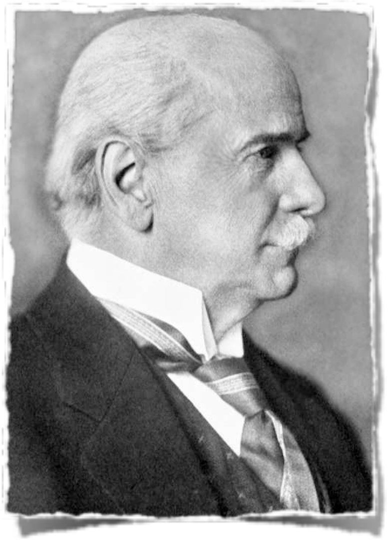

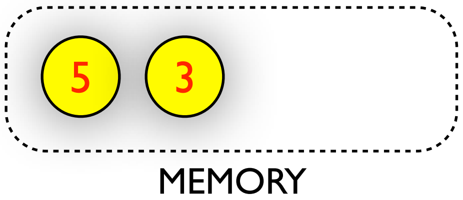

- At each step, user inserts a value into the memory or asks that the *smallest* value is extracted:

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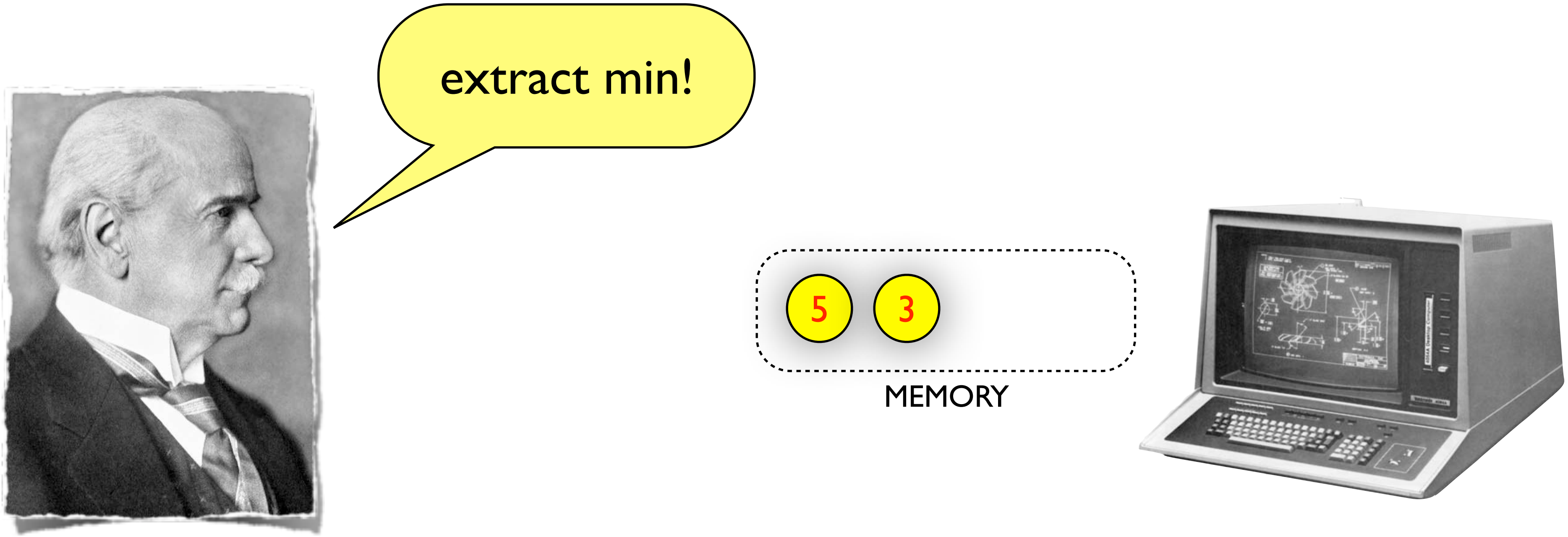

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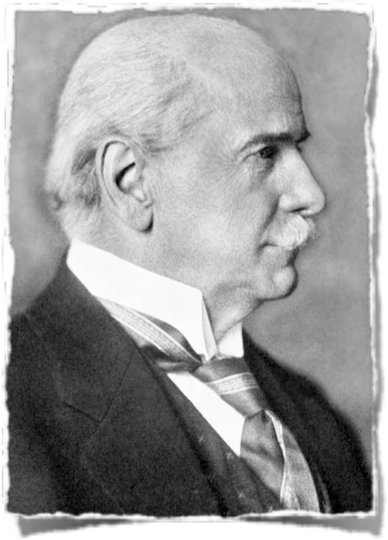

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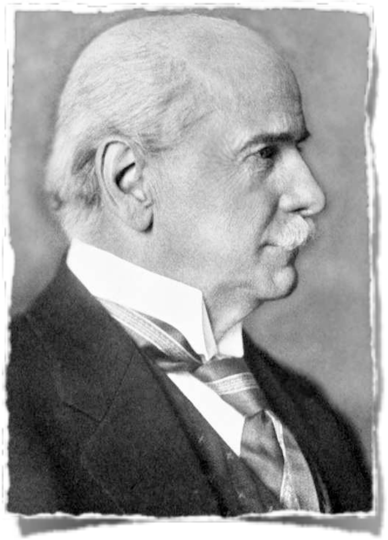

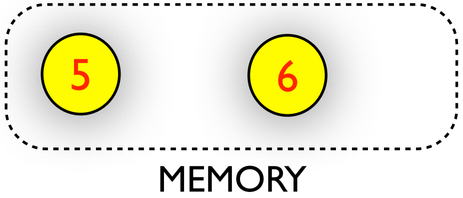

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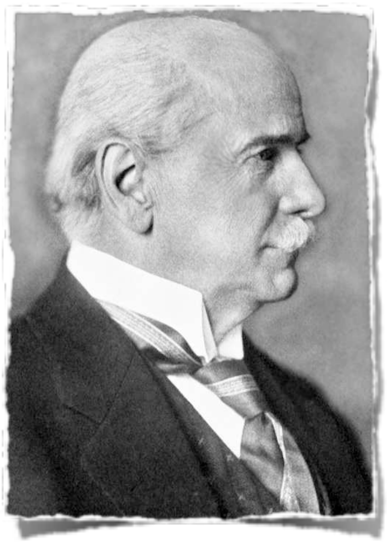

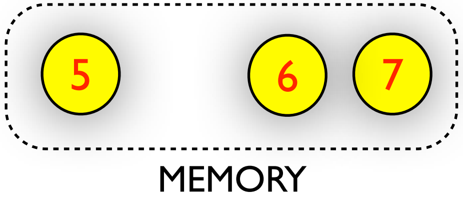

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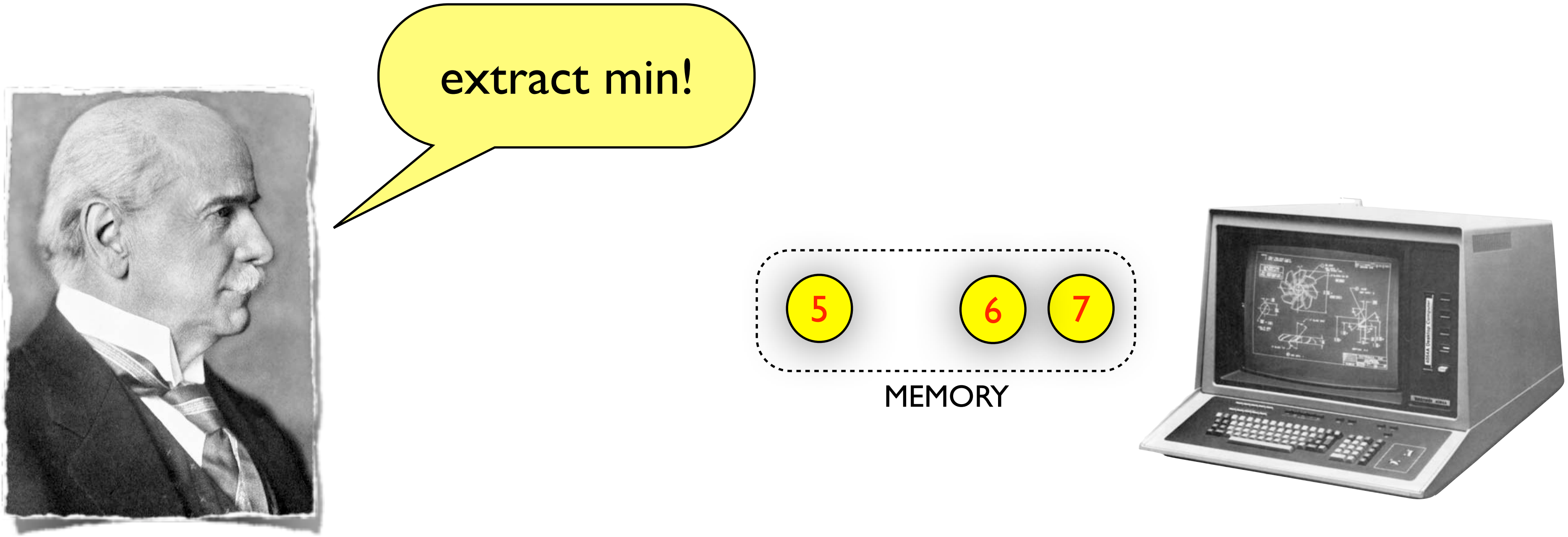

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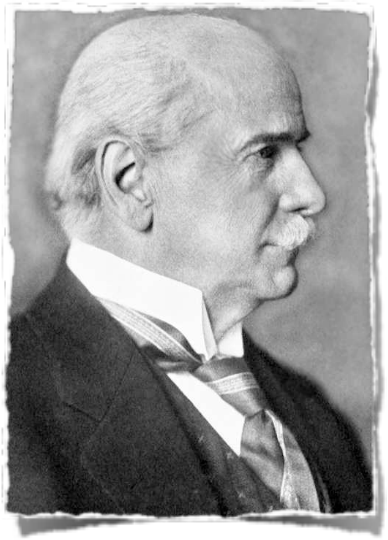

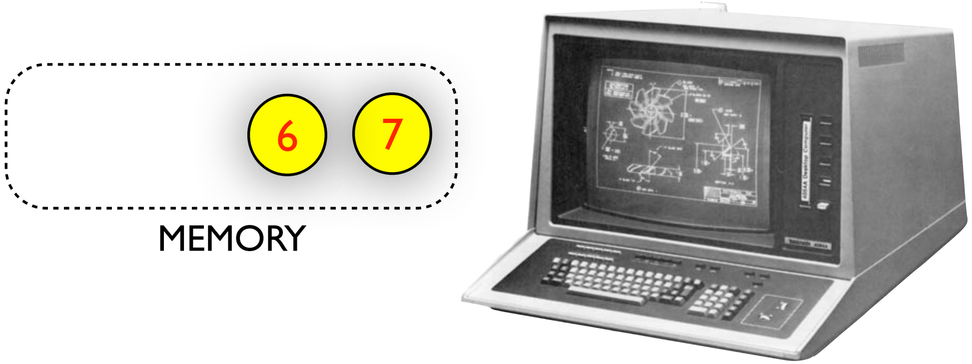

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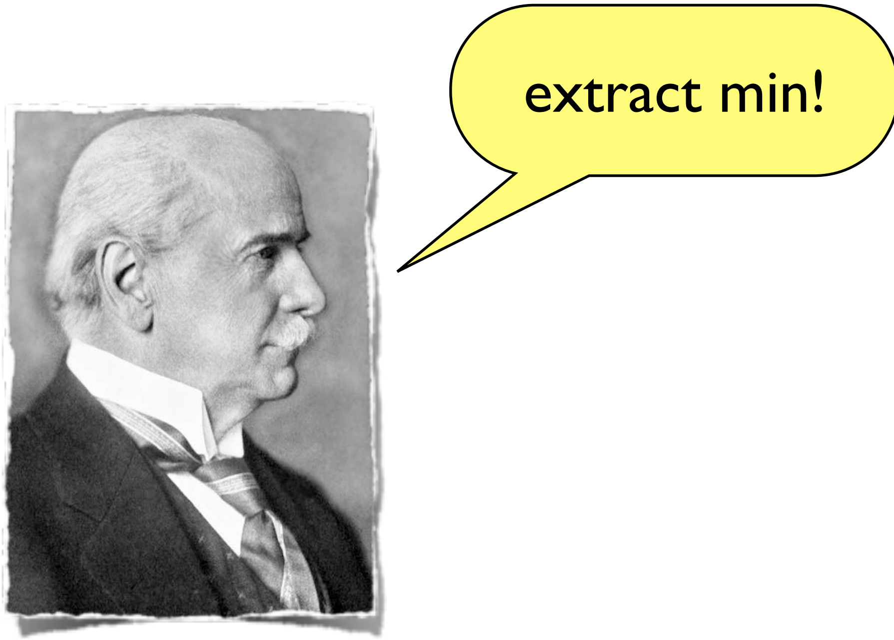

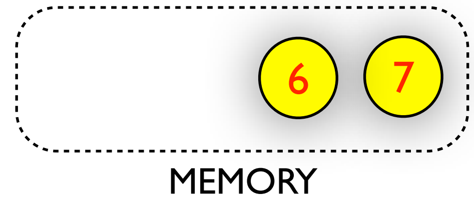

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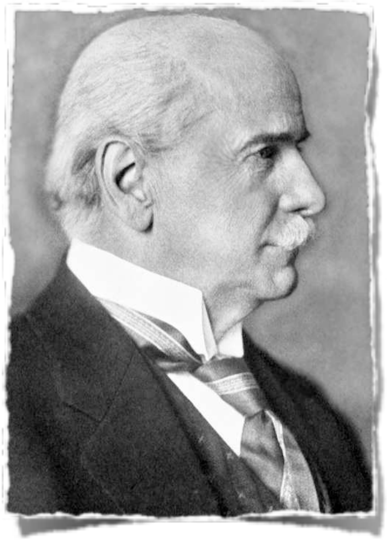

MEMORY

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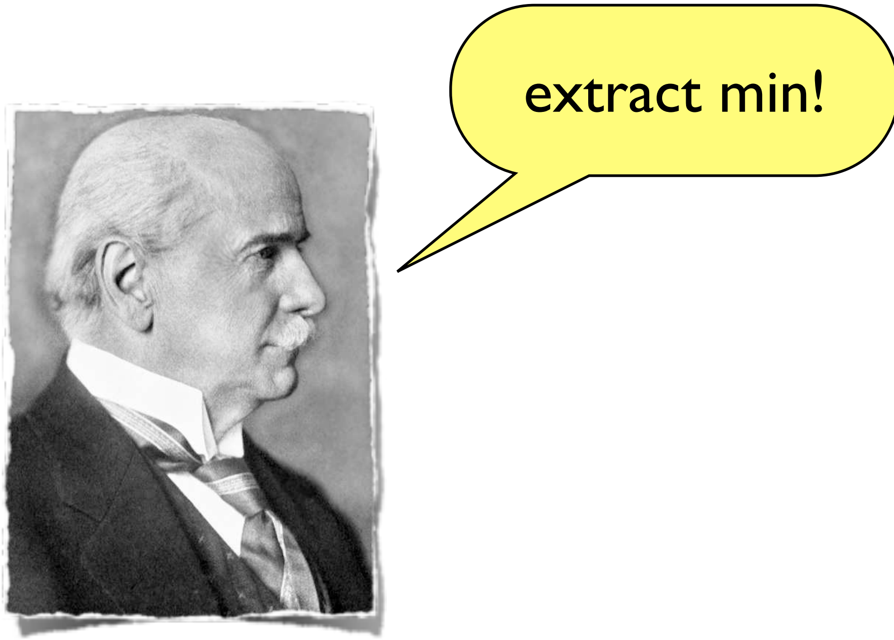

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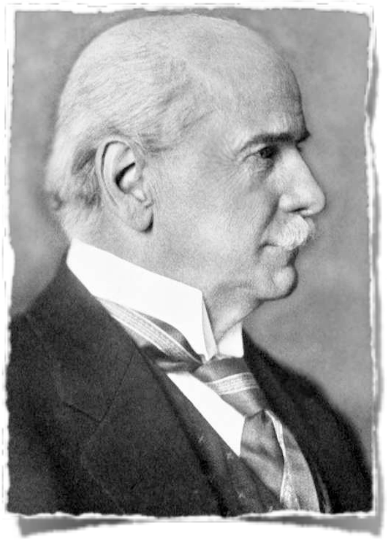

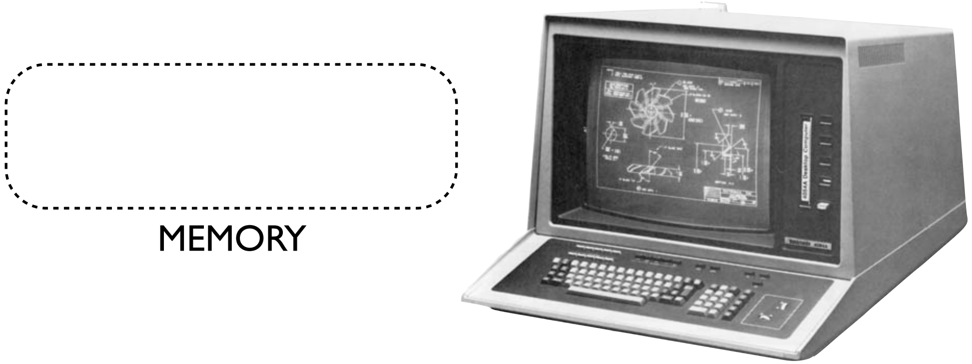

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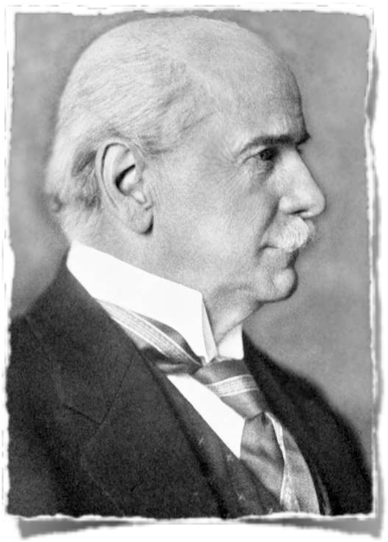

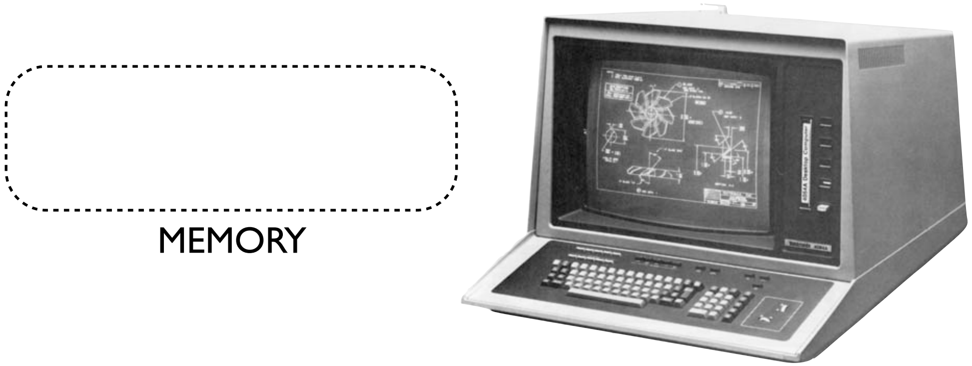

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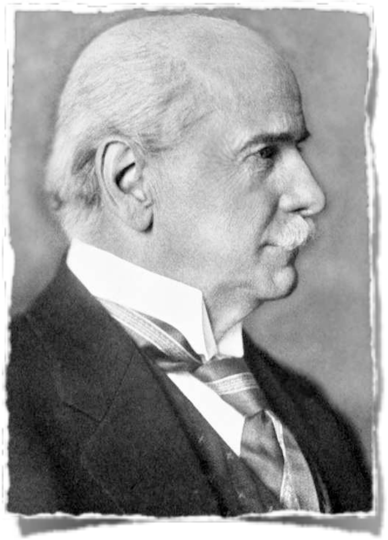

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- *Motivation:* Want to use cheap commodity hardware.

PQ Verification

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! *For this talk:* Assume inserted elements are distinct and that inserts come before their corresponding extract. I.e., we're trying to identify the following *bad pattern:*

ins(u) .... ext(v) ... ext(u) for some $u < v$

Epochs and Local Bad Patterns…

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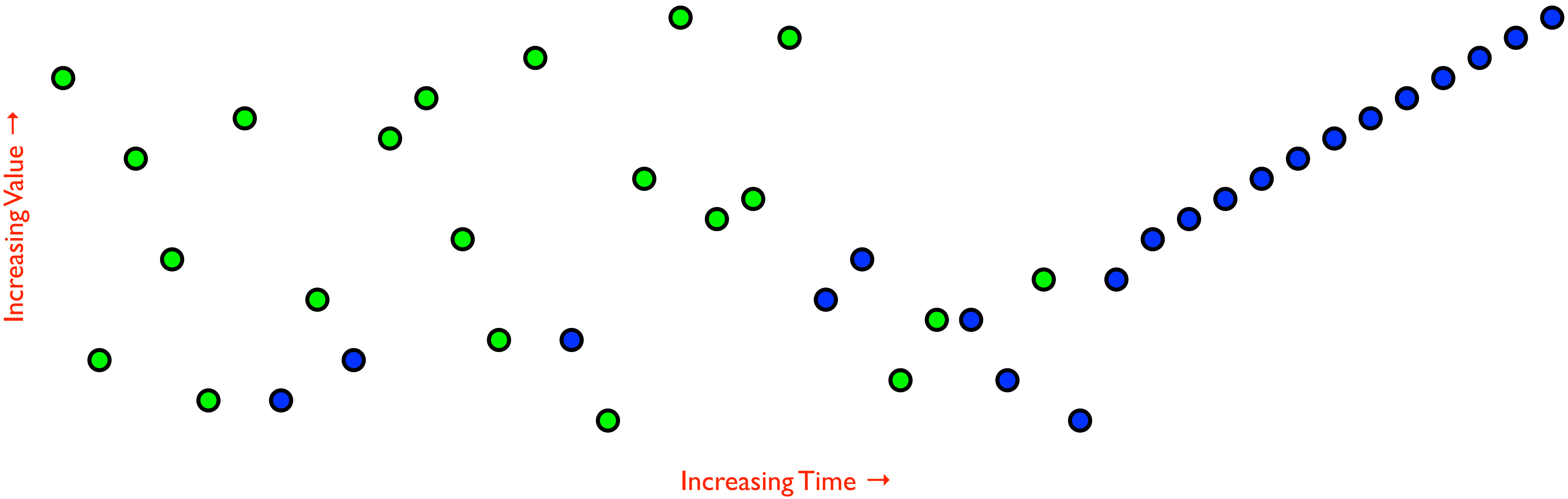

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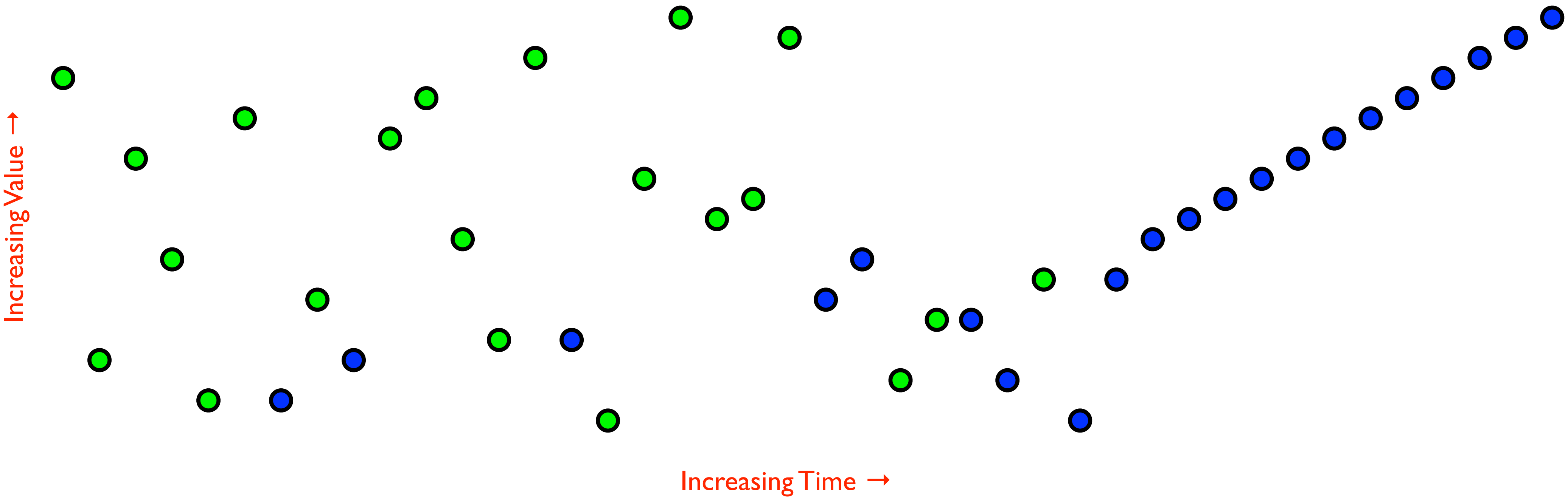

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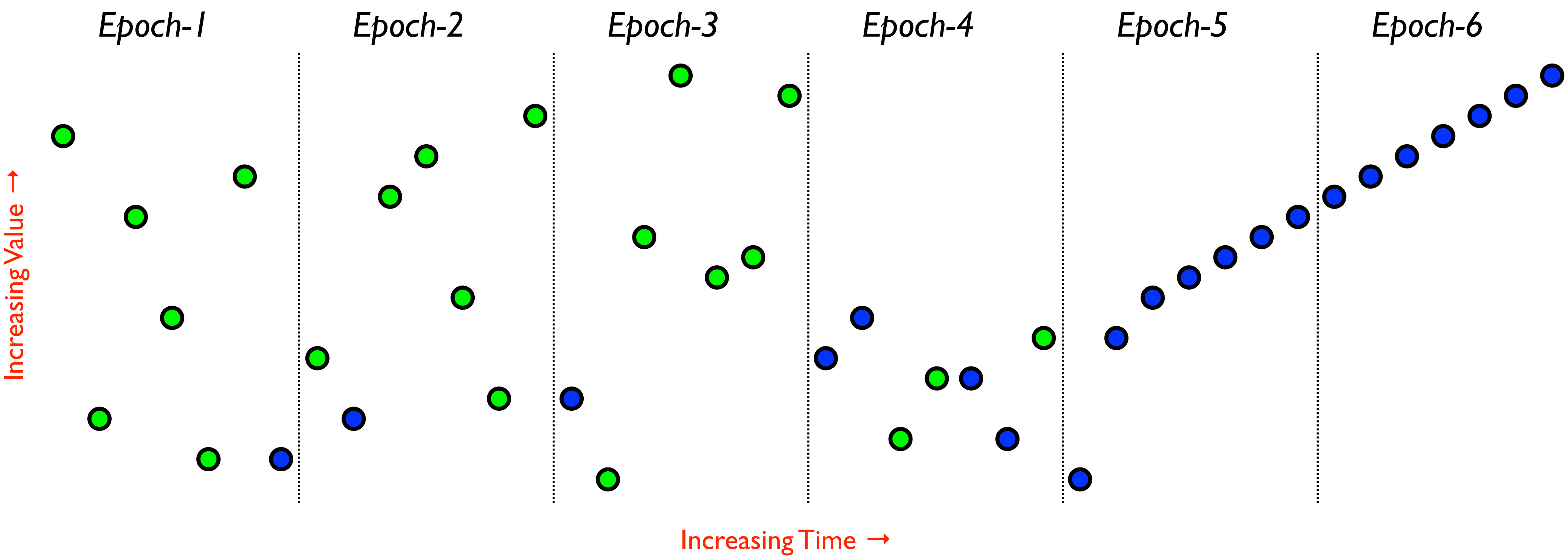

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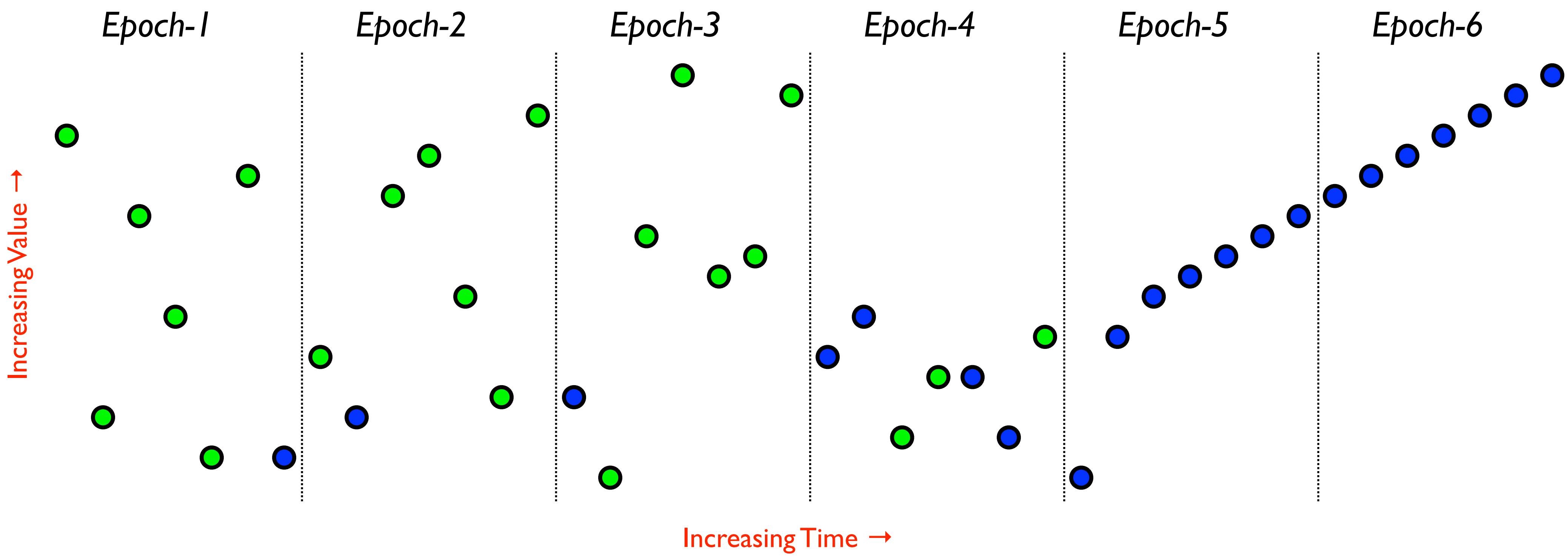

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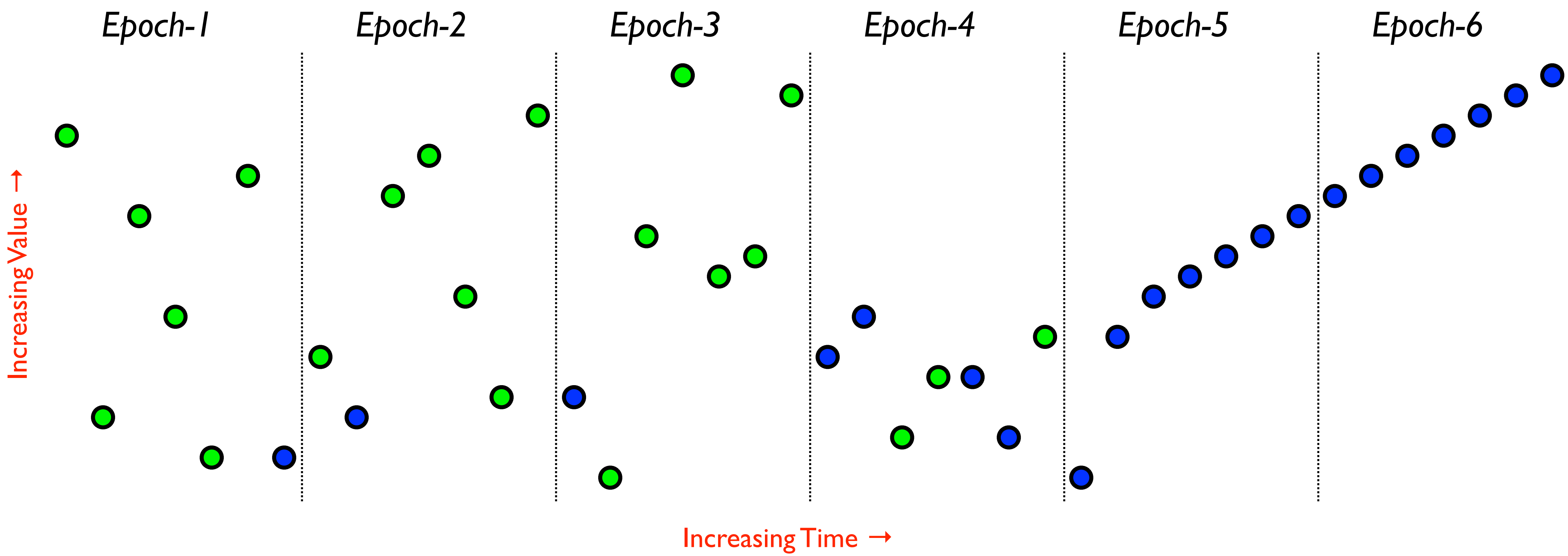

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- Using $O(\sqrt{N})$ space, we can buffer each epoch and check for local bad patterns.

Catching Long-Range Bad Patterns… 1/2

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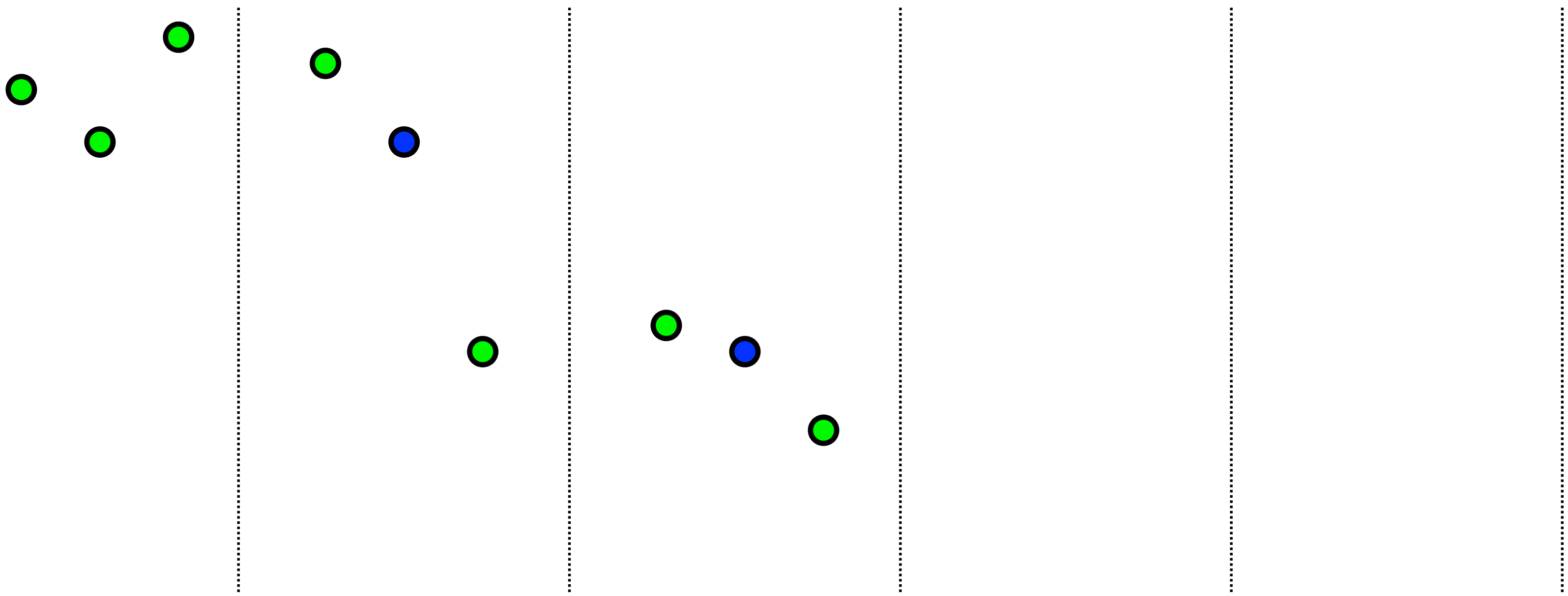

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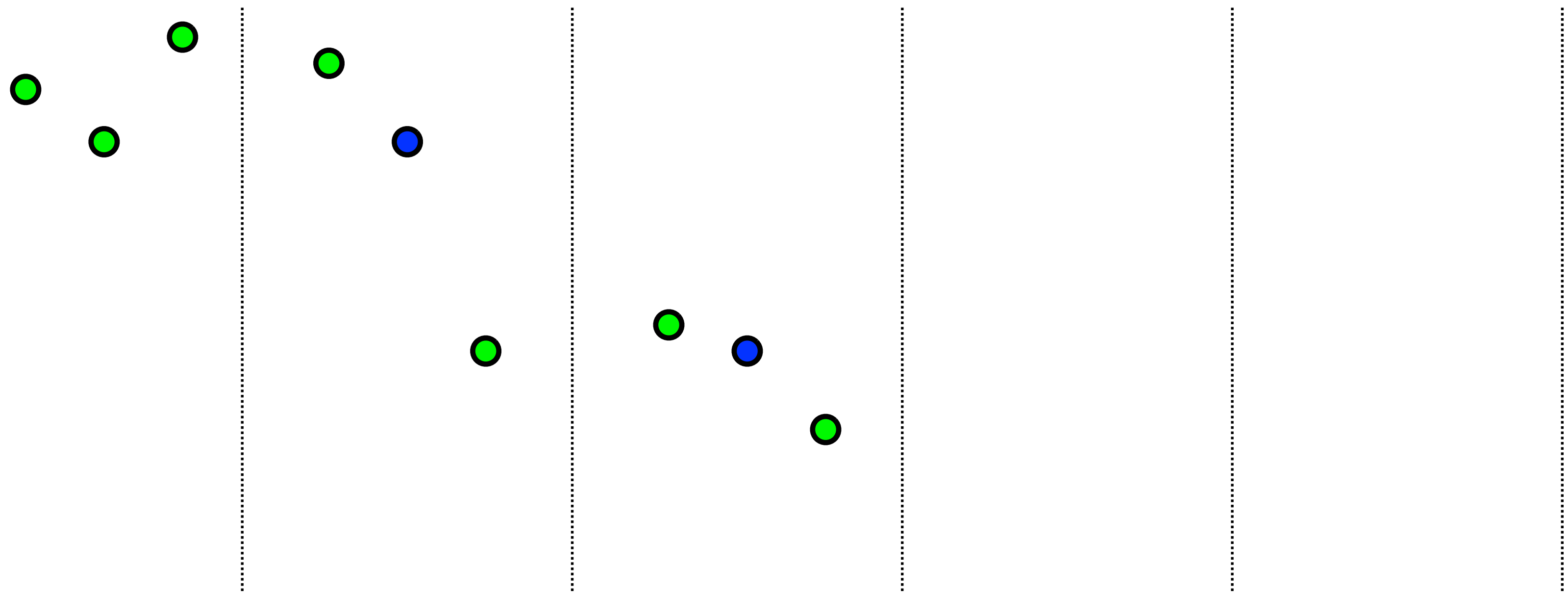
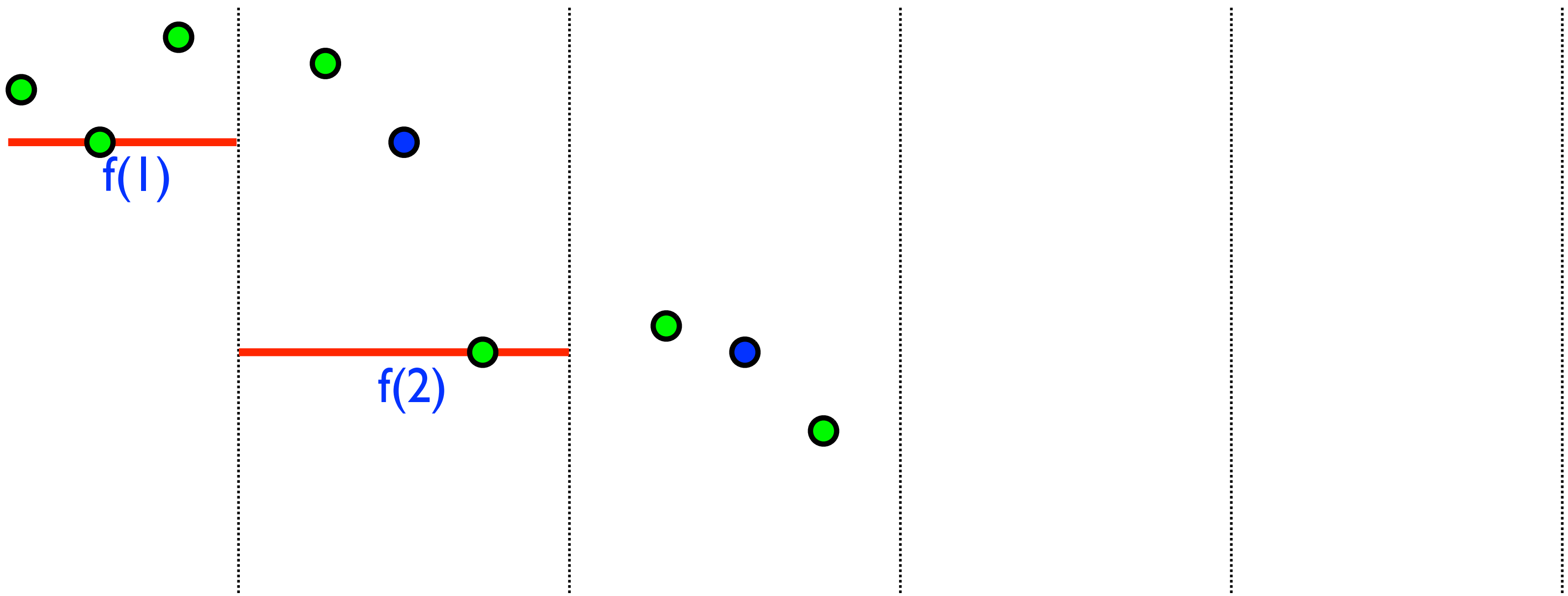


- Maintain the max value extracted between end of i-th epoch and *current time*. Call it f(i).

Catching Long-Range Bad Patterns... 1/2



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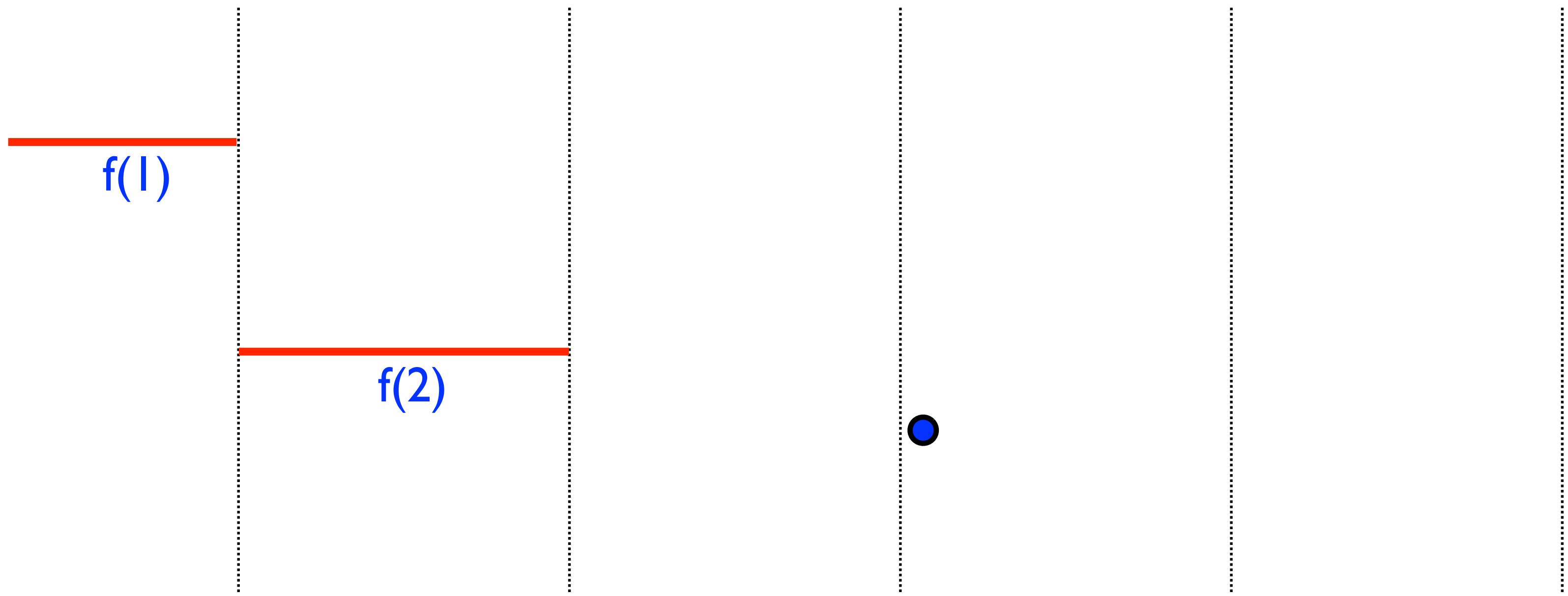

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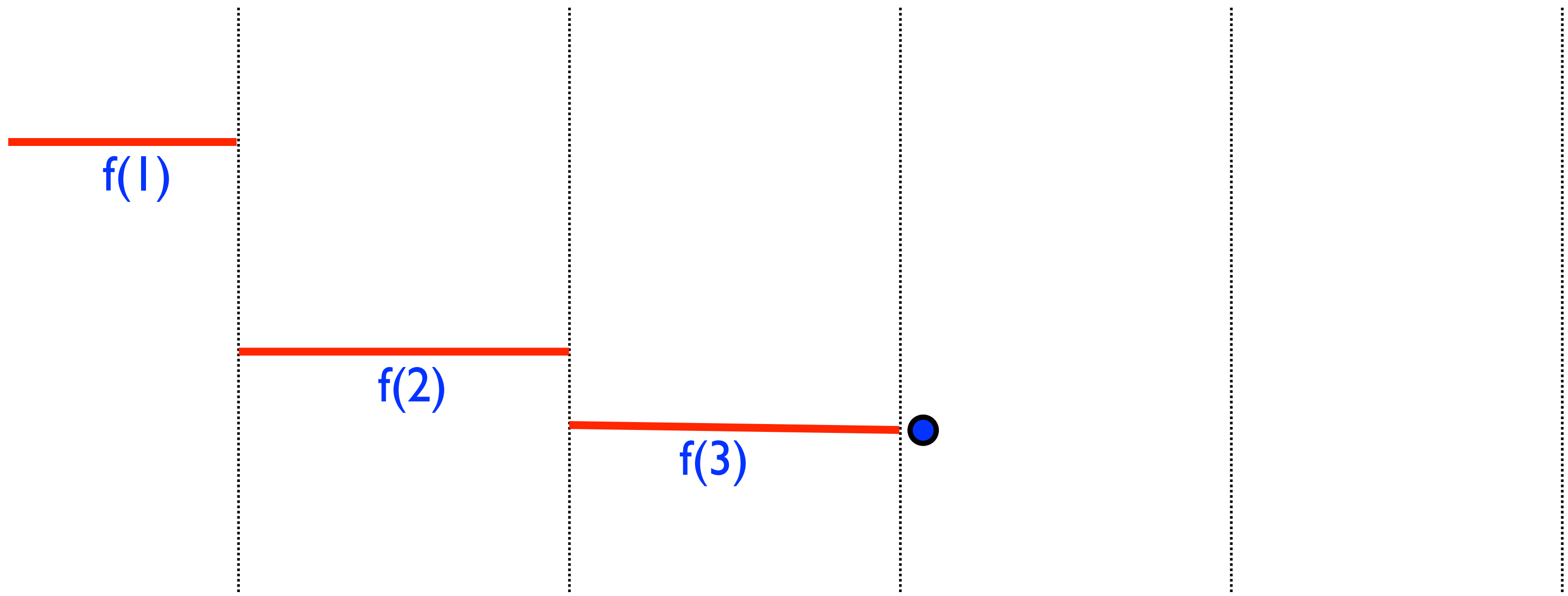

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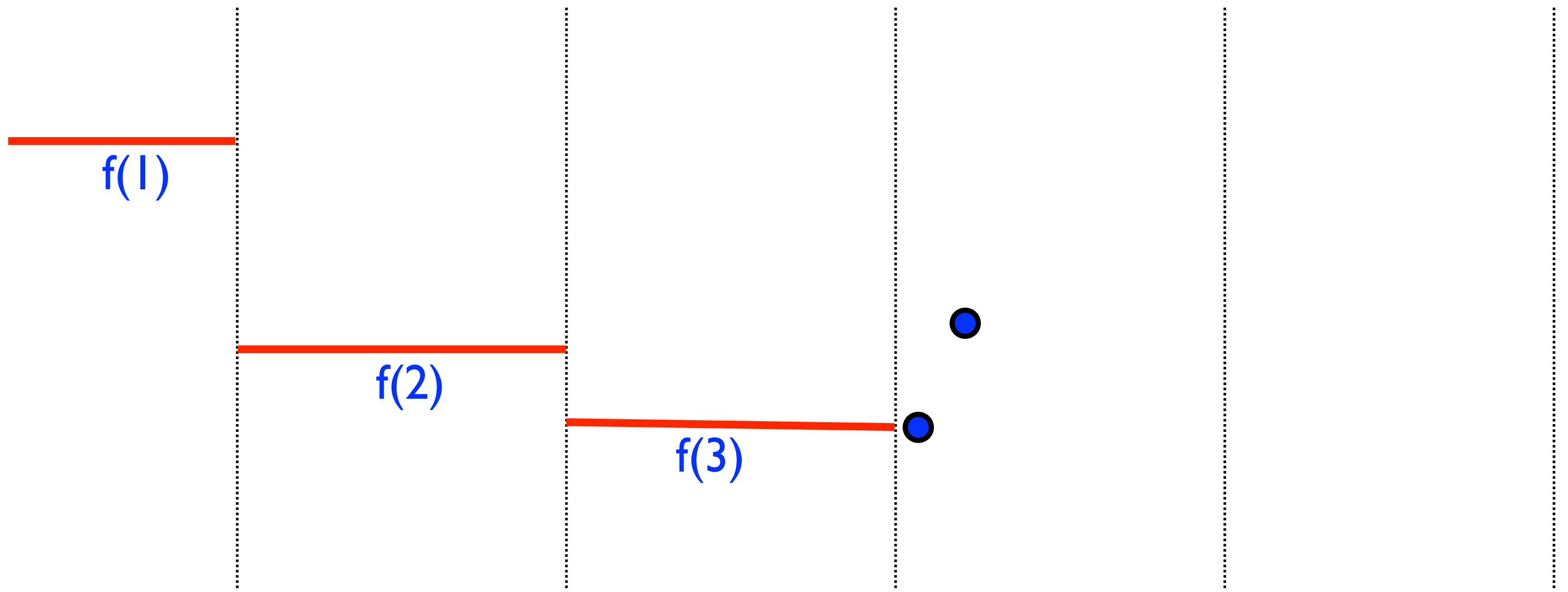

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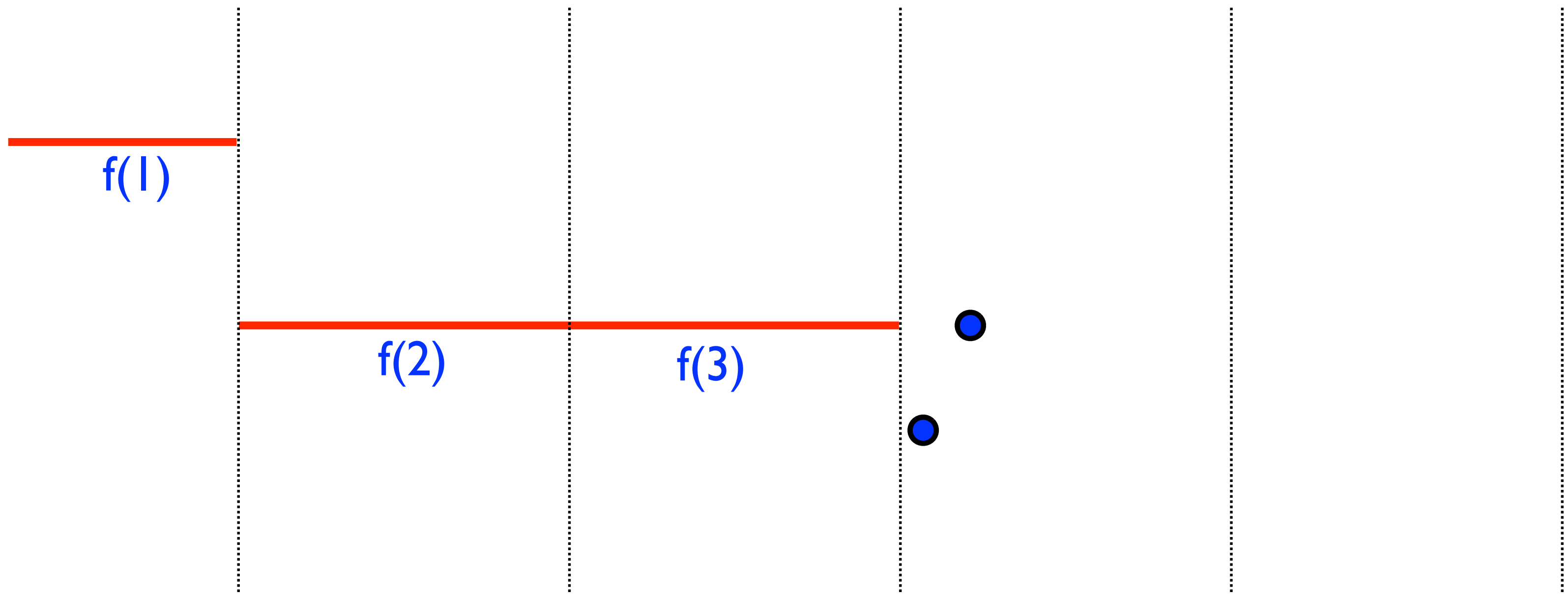

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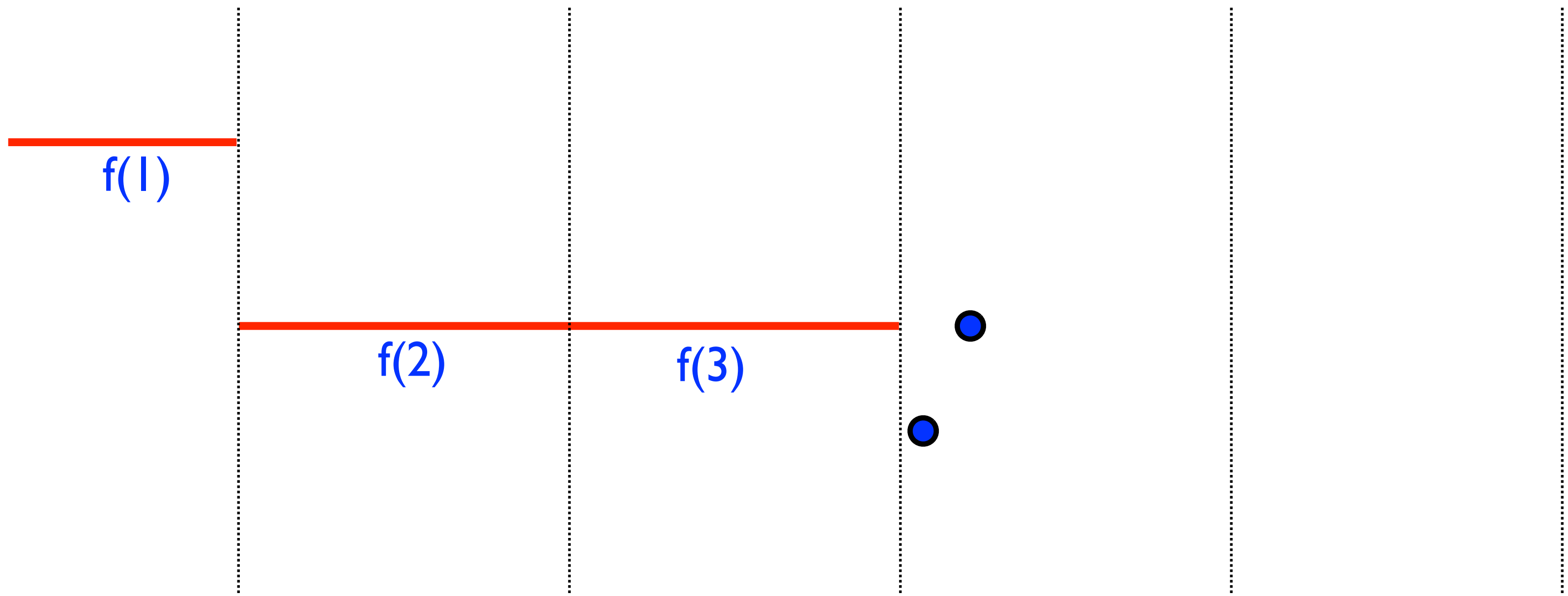

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- *Defn:* Each ins(u) or ext(u) is *adopted* by earliest epoch k with f(k)≤u.

Catching Long-Range Bad Patterns... 2/2

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- *Lemma:* If ins(u) ... ext(v) ... ext(u) is a long-range bad pattern then ins(u) and ext(u) are adopted by different epochs.

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- *Lemma:* If there are no bad patterns, every ins(u) and ext(u) pair get adopted by the same epoch.
- *Algorithm:* Using fingerprints to check: for each epoch k

$$\{u : \text{ins}(u) \text{ adopted by } k\} = \{u : \text{ext}(u) \text{ adopted by } k\}.$$

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- *Extensions:* Sub-linear space streaming recognition of other data structures like stacks, double-ended queues...