CMPSCI 711: More Advanced Algorithms
Section 4-1: Sequences

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Outline

Sequences and Strings

Longest Increasing Subsequence
Bracket Matching

- Consider a length $m$ stream of round and square brackets, e.g.

  \[ [ ] ( [ [ ] ] ) ( [ ] ) \]

- Say sequence is *well-balanced* if corresponding open and close brackets are of the same type.

- Can we recognize a well-balanced sequence in $O(\sqrt{n})$ space?
First Part of Result

Say a sequence is **t-turn** if there are \( t \) open brackets immediately followed by a close bracket. E.g., following sequence is 3-turn:

\[
[ ] ( [ [ ] ] ( [ ] ) )
\]

Helpful to consider sequence in 2D representation where *height* corresponds to excess of open brackets amongst preceding brackets.

\[
[ ] [ ]
[ ] ( )
[ ] ( )
\]

Using \( O(t) \) memory, it’s possible to determine height \( h \) of each new bracket and whether it’s the 1st, 2nd, \ldots \ bracket at this height.

As we read each symbol \( \alpha \), generate updates to \( x, y \in \{0, 1, 2\}^{t \times m} \)

\[
x_{h,i} \leftarrow x_{h,i} + \begin{cases} 
1 & \text{if } \alpha = "(" \\
2 & \text{if } \alpha = "["
\end{cases} \quad y_{h,i} \leftarrow y_{h,i} + \begin{cases} 
1 & \text{if } \alpha = ")" \\
2 & \text{if } \alpha = "]\"
\]

if \( \alpha \) is the \( i \)-th bracket at height \( h \). Then it suffices to check whether \( x = y \) and this can be done by sketching or finger-printing.
If we can ensure $t = O(\sqrt{n})$ the desired result follows.

Buffer length-$\sqrt{n}$ segments and perform all cancelations within each segment before passing it to the $t$-turn algorithm. E.g.,

\[
\begin{array}{c}
[ & ] ( [ [ ] ] [ ] ) ( [ & ] ) \\
\end{array}
\]

becomes

\[
\begin{array}{c}
( [ & ] [ & ] ) ( [ & ] ) \\
\end{array}
\]
Outline

Sequences and Strings

Longest Increasing Subsequence
Length of Longest Increasing Subsequence

- **Goal:** Estimate the length of the longest increasing subsequence.
- **Idea:** For $\ell \geq 0$, maintain

  $$A[\ell] = \text{smallest value that ends a length } \ell \text{ increasing sequence so far}$$

  - Initially, $A[0] = -\infty$ and $A[\ell] = \infty$ for all $\ell > 0$
  - If the new element is $v$, find smallest $\ell$ such that $A[\ell] \leq v$ and set

    $$A[\ell + 1] \leftarrow v$$