Question 1. In this question, we consider two polynomial-time reductions and an approximation algorithm that are in the textbook but that we didn’t go through in class.

a) Consider reduction from 3-SAT to Vertex-Cover in Section 7.3.4. Consider the 3-SAT formula \((x_1 \lor x_2 \lor x_3) \land (x_3 \lor x_4 \lor x_5) \land (\bar{x}_3 \lor x_4 \lor x_6)\). The reduction generates a graph based on this formula. How many edges does this graph have?

b) Consider reduction from 3-SAT to Subset-Sum in Section 7.3.5. Consider the 3-SAT formula \((x_1 \lor x_2 \lor x_3) \land (x_1 \lor \bar{x}_2 \lor \bar{x}_3)\). The reduction generates 11 integers based on this formula. What value do you get when you add them all together?

c) Read Section 8.6.1. Suppose an instance of Knapsack has \(n = 1024\) items where the value of the \(i\)th item is \(v_i = 2^i\). Let \(\epsilon = 1/16\). How many items will there be with non-zero values after the values get reduced as described in the algorithm?

Question 2. Consider a graph \(G = (V, E)\) that consists of four groups of nodes \(V = V_1 \cup V_2 \cup V_3 \cup V_4\) where \(|V_1| = 50, |V_2| = 100, |V_3| = 50, |V_4| = 49\). Every node in \(V_4\) has exactly two neighbors and both of these are also in \(V_4\). Every other node has degree 3 and every edge not incident to a node in \(V_4\) has exactly one endpoint in \(V_2\). For each of the following questions, either determine the value of the quantity or determine that it is not uniquely determined by the above information.

a) What is the minimum number of colors required to color the nodes such that the endpoints of every edge get different colors?
b) What is the number of edges?
c) How many connected components does the graph have?
d) What is the size of the maximum matching?
e) What is the size of the minimum vertex cover?
f) What is the size of the maximum independent set?