CMPSCI 311: Introduction to Algorithms

Lecture 5: Greedy Algorithms

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Problem 1: Interval Scheduling

- In the 80s, your only opportunity to watch a specific TV show was the time it was broadcast. Unfortunately, on a given night there might be multiple shows that you want to watch and some of the broadcast times overlap.
- You want to watch the highest number of shows. Which subset of shows do you pick?
- *Notation:* Let show j start at time s_j and finish at time f_j and we saw two shows are *compatible* if they don't overlap.
- Assume you like all shows equally, you only have one TV, and you need to watch shows in their entirety.

Greedy Algorithms

- Main idea in greedy algorithms is to sort the shows in some "natural order". Then consider the shows in this order and add a show to your list if it's compatible with the shows already chosen.
- What's a "natural order"?
  - *Start Time*: Consider shows in ascending order of s_j.
  - *Finish Time*: Consider shows in ascending order of f_j.
  - *Shortest Time*: Consider shows in ascending order of $f_j - s_j$.
  - *Fewest Conflicts*: Let c_j be number of shows which overlap with show j. Consider shows in ascending order of c_j.
- Unfortunately, not all of these approaches are going to maximize the number of shows you could watch.
- But, we'll show that considering the shows in order of the earliest finish time, maximizes the number of shows.

Ordering by Finish Time is gives an optimal answer: Part 1

- To simplify the notation assume $f_1 < f_2 < f_3 \ldots$
- Suppose the earliest-finish-time-ordering approach picks shows

$$A = \{i_1, i_2, \ldots, i_k\} \quad \text{where } i_1 < i_2 < \ldots$$

- For the sake of contraction suppose there's a set of $k' > k$ compatible shows

$$B = \{j_1, j_2, j_3 \ldots, j_{k'}\} \quad \text{where } j_1 < j_2 < \ldots$$

  - If there's more than one subset of k' compatible shows, pick the subset with $i_1 = j_1, \ldots, i_r = j_r$ for the max value of r.
  - Note that $i_{r+1} \neq j_{r+1}$ and $k \geq r + 1$ since the greedy algorithm could have picked show j_{r+1} after show i_r.

Ordering by Finish Time is gives an optimal answer: Part 2

- But consider the schedule formed from B by switched i_{r+1} with j_{r+1}:

$$C = \{j_1, j_2, \ldots, j_r, i_{r+1}, j_{r+2}, \ldots, j_{k'}\}$$

- C is also compatible:
  - i_{r+1} doesn't overlap with $\{j_1, \ldots, j_r\}$
  - Because i_{r+1} finishes before j_{r+1}, we know i_{r+1} doesn't overlap with $\{j_{r+1}, \ldots, j_{k'}\}$.
- But C shares more than the first r shows in common with B. This contradicts assumption B was subset of k' compatible shows with the most initial shows in common with A.

Problem 2: Interval Partitioning

- Suppose you are in charge of UMass classrooms.
- There are n classes to be scheduled on a Monday where class j starts at time s_j and finishes at time f_j
- Your goal is to schedule *all* the classes such that the minimum number of classrooms get used throughout the day. Obviously two classes that overlap can't use the same room.

Possible Greedy Approaches

- Suppose the available classrooms are numbered $1, 2, 3, \ldots$
- We could run a greedy algorithm...consider the lectures in some natural order, and assign the lecture to the classroom with the smallest numbered that is available.
- Continued next time...