Outline

1. Game Theory

2. Non Zero-Sum Games and Nash Equilibrium
Example: Two-finger Morra

- Alice and Bob play a game
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Obviously if Bob always plays the same number, Alice can take advantage of this. What if Bob plays different numbers with different probabilities?
Suppose Bob plays “1” with prob. $q$ and “2” with prob. $1 - q$
Analysis of Two-finger Morra (1/2)

- Suppose Bob plays “1” with prob. $q$ and “2” with prob. $1 - q$
- If Alice plays “1” then Alice has expected return

$$2q - 3(1 - q) = 5q - 3$$
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- If Alice plays her best option, she expects to get
  
  $$\max(5q - 3, 4 - 7q)$$

- Hence, if Bob sets $q = 7/12$ he still expects to win at least 1/12
Analysis of Two-finger Morra (2/2)

- Suppose Alice plays “1” with prob. $p$ and “2” with prob. $1 - p$

- If Bob plays “1” then Bob expects to get $-2p + 3(1 - p) = 3 - 5p$

- If Bob plays “2” then Bob expects to get $3p - 4(1 - p) = 7p - 4$

- If Bob plays his best option, he expects to get $\max(3 - 5p, 7p - 4)$

- Hence, if Alice sets $p = \frac{7}{12}$ she still expects to lose at most $\frac{1}{12}$

Conclusion:

Hence, best strategy for both players is play 1 finger with probability $\frac{7}{12}$ and 2 fingers with probability $\frac{5}{12}$. 
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Hence, if Alice sets \( p = \frac{7}{12} \) she still expects to lose at most \( \frac{1}{12} \)

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**Conclusion:** Hence, best strategy for both players is play 1 finger with probability 7/12 and 2 fingers with probability 5/12.
### Zero-Sum Games

**Definition**

A *two-player, simultaneous-move, zero-sum game* consists of a set of $k$ options for player $A$, a set of options $\ell$ for player $B$, and a $k \times \ell$ payoff *matrix* $P$. If $A$ is chooses option $i$ and $B$ chooses option $j$ then $A$ gets $P_{ij}$ and $B$ gets $-P_{ij}$.

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<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
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<tbody>
<tr>
<td>A</td>
<td>1</td>
<td>+2</td>
</tr>
<tr>
<td>B</td>
<td>2</td>
<td>+3</td>
</tr>
<tr>
<td></td>
<td>4</td>
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Definition

If a player picks one of their options, we call it a pure strategy. If they pick a distribution over their options, we call it a mixed strategy. If one option is better than other no matter what the other player does, we say the first strategy dominates the second.
Example: Three-finger Morra

- Alice and Bob play a game
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<td>−5</td>
</tr>
<tr>
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<td>+4</td>
<td>−5</td>
<td>+6</td>
</tr>
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Suppose A plays “1” with probability $r$, “2” with probability $s$, and “3” with probability $1 - r - s$.
Suppose A plays “1” with probability $r$, “2” with probability $s$, and “3” with probability $1 - r - s$

If B plays “1” then A’s expected reward is

\[2r - 3s + 4(1 - r - s) = 4 - 2r - 7s\]
Suppose A plays “1” with probability $r$, “2” with probability $s$, and “3” with probability $1 - r - s$

If B plays “1” then A’s expected reward is

$$2r - 3s + 4(1 - r - s) = 4 - 2r - 7s$$

If B plays “2” then A’s expected reward is

$$-3r + 4s - 5(1 - r - s) = -5 + 2r + 9s$$
Suppose A plays “1” with probability $r$, “2” with probability $s$, and “3” with probability $1 - r - s$.

If B plays “1” then A’s expected reward is

$$2r - 3s + 4(1 - r - s) = 4 - 2r - 7s$$

If B plays “2” then A’s expected reward is

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If B plays “3” then A’s expected reward is

$$4r - 5s + 6(1 - r - s) = 6 - 2r - 11s$$
Analysis of Three-finger Morra (1/2)

- Suppose A plays “1” with probability $r$, “2” with probability $s$, and “3” with probability $1 - r - s$
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- Hence, for $r = 1/4$, $s = 1/2$, A gets expected return at least 0
Suppose B plays “1” with probability $r$, “2” with probability $s$, and “3” with probability $1 - r - s$
Analysis of Three-finger Morra (2/2)

- Suppose B plays “1” with probability $r$, “2” with probability $s$, and “3” with probability $1 - r - s$
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Hence, best strategy for each player is show 1 finger with probability $1/4$ and 2 fingers with probability $1/2$. 
Suppose B plays “1” with probability $r$, “2” with probability $s$, and “3” with probability $1 - r - s$

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Hence, best strategy for each player is show 1 finger with probability $1/4$ and 2 fingers with probability $1/2$. 

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Outline

1. Game Theory

2. Non Zero-Sum Games and Nash Equilibrium
Prisoner’s Dilemma

Two prisoners are being held pending trial for a crime they are alleged to have committed. The prosecutor offers each a deal:

“Give evidence against your partner and you'll go free, unless your partner also confesses. If both confess, both get 5 year sentences. If neither confess, both get 1 year sentences. If you don’t confess but your partner does, you get 10 years!”
Two prisoners are being held pending trial for a crime they are alleged to have committed. The prosecutor offers each a deal:

“Give evidence against your partner and you’ll go free, unless your partner also confesses. If both confess, both get 5 year sentences. If neither confess, both get 1 year sentences. If you don’t confess but your partner does, you get 10 years!”

Can represent this as a game but it’s not zero-sum:

<table>
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<tr>
<th></th>
<th>B Confesses</th>
<th>B Stays Mute</th>
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<tbody>
<tr>
<td>A Confesses</td>
<td>−5, −5</td>
<td>0, −10</td>
</tr>
<tr>
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<td>−10, 0</td>
<td>−1, −1</td>
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In each entry, the first number is A’s reward and the second number of B’s reward.
Nash Equilibrium

**Definition**

A Nash Equilibrium is a set of strategies for each player where no change by one player alone can improve his outcome.
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For the prisoners dilemma the unique Nash Equilibrium is that both prisoners confess.

**Theorem (Nash)**

*Every game where each player has a finite number of options, has at least one Nash equilibrium.*
Hawks and Doves

- Two birds meet over a piece of food and have to decide whether to act aggressive (hawkish) or passive (dovish).

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No Nash Equilibrium where both players play same pure strategies:
- If A and B are Hawks, both would prefer to switch to Doves.
- If A and B are Doves, both would prefer to switch to Hawks.
- A plays Hawk and B plays Dove is a Nash Equilibria and vice versa.
Hawks and Doves

- Two birds meet over a piece of food and have to decide whether to act aggressive (hawkish) or passive (dovish)
  - If a hawk meets a dove, the hawk gets the food worth 50 points

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  - A plays Hawk and B plays Dove is a Nash Equilibrium and vice versa
A Mixed Strategy Nash Equilibrium for Hawks and Doves

Suppose Alice plays Hawk with probability $0 < q < 1$ and Bob plays Hawk with probability $0 < p < 1$. Alice's expected reward is

$$-25pq + 50q(1-p) + 15(1-p)(1-q)$$

When can Alice not improve by changing $q$?

When $p = \frac{7}{12}$.

Bob's expected reward is

$$-60pq + 35p + 15 - 15q$$

When can Bob not improve by changing $p$?

When $q = \frac{7}{12}$.

Hence $p = q = \frac{7}{12}$ is only Nash Equilibrium with mixed strategies.
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  = q(35 - 60p) + 15 - 15p
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When can Alice not improve by changing $q$?
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