Outline

1. Review
2. Conditional PMFs
3. Expectation and Variance
4. More Markov and Chebyshev
Expectation and Variance Review

- The expected value $E[X]$ of a random variable $X$ is a probability-weighted average of the possible values of $X$:

$$E[X] = \sum_k k P(X = k)$$
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$$E[X] = \sum_{k} k P(X = k)$$

If $X$ is a random variable and $f : \mathbb{R} \rightarrow \mathbb{R}$ then $Y = f(X)$ is also a random variable with expectation

$$E(Y) = \sum_{k} f(k)P(X = k)$$
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The variance is quantifies how close to $\mu = E[X]$ we expect $X$ to be:

$$\text{var}(X) = E[(X - \mu)^2]$$
The expected value \( E[X] \) of a random variable \( X \) is a probability-weighted average of the possible values of \( X \):

\[
E[X] = \sum_k k P(X = k)
\]

If \( X \) is a random variable and \( f : \mathbb{R} \to \mathbb{R} \) then \( Y = f(X) \) is also a random variable with expectation

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E(Y) = \sum_k f(k)P(X = k)
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The variance is quantifies how close to \( \mu = E[X] \) we expect \( X \) to be:

\[
\text{var}(X) = E[(X - \mu)^2] = E[X^2] - \mu^2.
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The variance is quantifies how close to $\mu = E[X]$ we expect $X$ to be:

$$\text{var}(X) = E[(X - \mu)^2] = E[X^2] - \mu^2.$$ 

The standard deviation of $X$ is $\sigma_X = \sqrt{\text{var}(X)}$. 
Markov and Chebyshev Bounds

- **Markov Bound:** For an non-negative random variable $X$,

$$P(X \geq c) \leq \frac{E(X)}{c}$$

- **Chebyshev Bound:** For a random variable $X$,

$$P(|X - E(X)| \geq c) \leq \frac{Var(X)}{c^2}$$
Multiple Random Variables

Consider two random variables, $X$ and $Y$ mapping from $\Omega$ to $\mathbb{R}$. For $i, j \in \mathbb{R}$, we can define the event $\{X = i, Y = j\} = \{X = i\} \cap \{Y = j\} = \{o \in \Omega | X(o) = i \text{ and } Y(o) = j\}$. The probabilities of these events give the joint PMF of $X$ and $Y$: $P(X = i, Y = j) = P(\{X = i, Y = j\})$. 


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$$P(X = i, Y = j) = P(\{X = i, Y = j\})$$
Tabular Representation of Joint PMFs

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- e.g., \( P(X = 2, Y = 3) = 0.1, \ P(X = 3, Y = 1) = 0, \ldots \)
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- e.g., $P(X = 2, Y = 3) = 0.1$, $P(X = 3, Y = 1) = 0$, ...
- Given the joint PMF, how can we work out $P(X = i)$ and $P(Y = j)$?
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- Given the joint PMF, how can we work out $P(X = i)$ and $P(Y = j)$?

\[
P(X = i) = \sum_j P(X = i, Y = j)
\]

\[
P(Y = j) = \sum_i P(X = i, Y = j)
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- If we start with the joint PMF of \( X \) and \( Y \), we refer to \( P(X) \) as the marginal PMF of \( X \) and \( P(Y) \) as the marginal PMF of \( Y \).
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**Conditional PMFs**

- **Conditional PMF of X given Y**:

  \[ P(X = i | Y = j) = P(\{X = i\} | \{Y = j\}) \].
Conditioning

- **Conditional PMF of** $X$ **given** $Y$:

  $$P(X = i | Y = j) = P(\{X = i\} | \{Y = j\}).$$

- **Compute** $P(X | Y)$ **using the definition of conditional probability**:

  $$P(X = i | Y = j) = \frac{P(X = i, Y = j)}{P(Y = j)}$$

  since for any two events $A, B$ we have $P(A | B) = \frac{P(A \cap B)}{P(B)}$. 
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- The conditional probability $P(X = i | Y = j)$ is the joint probability $P(X = i, Y = j)$ normalized by the marginal $P(Y = j)$.
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- The conditional probability $P(X = i | Y = j)$ is the joint probability $P(X = i, Y = j)$ normalized by the marginal $P(Y = j)$.

- An equivalent definition of independence is $X$ and $Y$ are independent if

for all $i, j$, $P(X = i | Y = j) = P(X = i)$
### Conditional PMFs

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Outline

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2. Conditional PMFs
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4. More Markov and Chebyshev
Given two random variables $X$ and $Y$ and a function $f : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$, 

$$Z = f(X, Y)$$

is a new random variable where

$$E(Z) = \sum_{x, y} f(x, y)P(X = x, Y = y)$$

and $var(Z) = E(Z^2) - (E(Z))^2$. 

Linearity of Expectation: If $Z = X + Y$,

$$E(Z) = E(X + Y) = E(X) + E(Y)$$

Linearity of Variance: If $Z = X + Y$,

$$var(Z) = var(X + Y) = var(X) + var(Y)$$

if $X$ and $Y$ are independent, i.e., for all $i, j$

$$P(X = i, Y = j) = P(X = i)P(Y = j)$$.
Functions of Two Random Variables

- Given two random variables $X$ and $Y$ and a function $f : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$, 
  
  $$Z = f(X, Y)$$

  is a new random variable where

  $$E(Z) = \sum_{x, y} f(x, y)P(X = x, Y = y)$$ and $\text{var}(Z) = E(Z^2) - (E(Z))^2$.

- **Linearity of Expectation**: If $Z = X + Y$, 

  $$E(Z) = E(X + Y) = E(X) + E(Y)$$
Functions of Two Random Variables

- Given two random variables $X$ and $Y$ and a function $f : \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$,
  
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  is a new random variable where

  \[ E(Z) = \sum_{x,y} f(x, y)P(X = x, Y = y) \text{ and } var(Z) = E(Z^2) - (E(Z))^2. \]

- **Linearity of Expectation:** If $Z = X + Y$,
  
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- **Linearity of Variance:** If $Z = X + Y$,
  
  \[ var(Z) = var(X + Y) = var(X) + var(Y) \]

  if $X$ and $Y$ are independent, i.e., for all $i, j$

  \[ P(X = i, Y = j) = P(X = i)P(Y = j). \]
**Linearity of Expectation**

**Lemma:** Given two random variables $X$, $Y$, and $Z = X + Y$ then

Linearity of Expectation

■ **Lemma:** Given two random variables $X$, $Y$, and $Z = X + Y$ then

\[
\]

■ **Proof:** Generalized expected value rule.

\[
E[Z] = \sum_{a} \sum_{b} (a + b) \cdot P(X = a, Y = b)
\]

\[
= \sum_{a} \sum_{b} a \cdot P(X = a, Y = b) + \sum_{a} \sum_{b} b \cdot P(X = a, Y = b)
\]

\[
= \sum_{a} a \sum_{b} P(X = a, Y = b) + \sum_{b} b \sum_{a} P(X = a, Y = b)
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\[
= \sum_{a} aP(X = a) + \sum_{b} bP(Y = b) = E(X) + E(Y)
\]
Lemma: If $X$ and $Y$ are independent then $E[XY] = E[X]E[Y]$:
**Lemma:** If $X$ and $Y$ are independent then $E[XY] = E[X]E[Y]$.

**Proof:**

\[
E[XY] = \sum_{a} \sum_{b} ab \cdot P(X = a, Y = b)
\]

\[
= \sum_{a} \sum_{b} ab \cdot P(X = a)P(Y = b)
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\[
= \sum_{a} a \cdot P(X = a) \cdot \sum_{b} b \cdot P(Y = b)
\]

\[
= E[X]E[Y]
\]
Lemma: If $X$ and $Y$ are independent then

$$\text{var}(X + Y) = \text{var}(X) + \text{var}(Y)$$
Variance of Sums of Random Variables

**Lemma:** If $X$ and $Y$ are independent then

$$\text{var}(X + Y) = \text{var}(X) + \text{var}(Y)$$

**Proof:**

$$\text{var}(X + Y) = E[(X + Y)^2] - E[X + Y]^2$$
$$= E[X^2 + 2XY + Y^2] - (E[X] + E[Y])^2$$
$$- (E[X]^2 + 2E[X]E[Y] + E[Y]^2)$$
$$= \text{var}(X) + \text{var}(Y)$$
Functions of Multiple Random Variables

Given random variables $X_1, X_2, \ldots, X_N$ and a function $f : \mathbb{R} \times \mathbb{R} \times \ldots \times \mathbb{R} \to \mathbb{R}$,

$$Z = f(X_1, X_2, \ldots, X_N)$$

is a new random variable.
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- **Linearity of Expectation:** If $Z = \sum_{i=1}^{N} X_i$,

$$E(Z) = E\left(\sum_{i=1}^{N} X_i\right) = \sum_{i=1}^{N} E(X_i)$$
Functions of Multiple Random Variables

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- **Linearity of Variance:** If $Z = \sum_{i=1}^{N} X_i$,

$$\text{var}(Z) = \text{var}(\sum_{i=1}^{N} X_i) = \sum_{i=1}^{N} \text{var}(X_i)$$

if $X_1, \ldots, X_N$ are pairwise independent, i.e., for all $a, b$

$$P(X_i = a, X_j = b) = P(X_i = a)P(X_j = b).$$
Example 1

Toss 12 fair six-sided dice. Let $X$ be the number of “1”s and let $Y$ be the number of “6”s. Are $X$ and $Y$ independent:

A: Yes
B: No
C: Can’t tell from the information given.
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Toss 12 fair six-sided dice. Let $X$ be the number of “1”s and let $Y$ be the number of “6”s. Are $X$ and $Y$ independent:

A: Yes
B: No
C: Can’t tell from the information given.

Answer is B since $P(X = 12, Y = 12) = 0 \neq P(X = 12)P(Y = 12)$. 
Example 2

Toss 12 fair six-sided dice. Let $X$ be the number of “1”s and let $Y$ be the number of “6”s. What is the expected value of $X$?

A: 0  
B: 1  
C: 2  
D: 3  
E: 6
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Toss 12 fair six-sided dice. Let $X$ be the number of “1”s and let $Y$ be the number of “6”s. What is the expected value of $X$?

A: 0  
B: 1  
C: 2  
D: 3  
E: 6

Answer is C since each of the 12 throws has a 1/6 of being a “1”.
Example 3

Toss 12 fair six-sided dice. Let $X$ be the number of “1”s and let $Y$ be the number of “6”s. What is the expected value of $X + Y$?

A: 0
B: 2
C: 4
D: 6
E: 12

Answer is C because $E(X + Y) = E(X) + E(Y) = 2 + 2 = 4$. 
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Secrets of the Markov Bound

- Markov Bound: For any non-negative random variable, \( P(X \geq c) \leq E(X)/c \)
Secrets of the Markov Bound

- Markov Bound: For any non-negative random variable,

\[ P(X \geq c) \leq \frac{E(X)}{c} \]

- For example, if \( E(X) = 10 \),

\[ P(X \geq 15) \leq \frac{2}{3} \]
Secrets of the Markov Bound

- Markov Bound: For any non-negative random variable,
  \[ P(X \geq c) \leq \frac{E(X)}{c} \]

- For example, if \( E(X) = 10 \),
  \[ P(X \geq 15) \leq 2/3 \]

- Can we infer an interesting upper bound about a lower tail, e.g.,
  \[ P(X \leq 5) \leq ?? \]
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- No! Consider a random variable of the form \( X = 10t \) with probability \( 1/t \) and \( X = 0 \) with probability \( 1 - 1/t \). Then, \( E(X) = 10 \) but

\[ P(X \leq 5) = 1 - 1/t \]

can be arbitrarily close to 1.
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and $Y \geq 0$. Then,

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$$P(X \leq 5) = P(Y \geq 10) \leq E(Y)/10 = 1/2$$
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- For example, if \( E(X) = 10 \) and \( var(X) = 2 \) then

\[
P(X \geq 15) = P(X \geq E(X) + 5) \leq P(|X - E(X)| \geq 5) \leq \frac{2}{25}
\]