CMPSCI 240: Reasoning Under Uncertainty
Mathematics Cheat Sheet

Closed-form summations

$$\sum_{i=1}^{n} c = cn \tag{1}$$

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2} \tag{2}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \tag{3}$$

$$\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4} \tag{4}$$

$$\sum_{i=0}^{n} ar^i = a\Big(\frac{1-r^{n+1}}{1-r}\Big) \tag{5}$$

$$\sum_{i=0}^{\infty} ar^i = \frac{a}{1-r}, r < 1 \tag{6}$$

Summation Operations

Summation is a linear operator, so the usual rules for linear operators apply. Remember that sums are inclusive, so their endpoints are included in the computation.

$$\sum_{i=1}^{n}(x_i + y_i) = \sum_{i=i}^{n} x_i + \sum_{i=1}^{n} y_i \tag{7}$$

$$\sum_{i=1}^{n}(x_i + y_i)^k = \sum_{i=1}^{n}\sum_{j=0}^{k}\binom{k}{j} x_i^{k-j} y_i^j \tag{8}$$

$$\sum_{i=i}^{n} cx_i = c\sum_{i=1}^{n} x_i \tag{9}$$

It is sometimes helpful to rewrite sums, especially if there is a particular sub-sum of interest:

$$\sum_{i=0}^{n} i = \sum_{i=0}^{k} i + \sum_{i=k+1}^{n} i \tag{10}$$

$$\sum_{i=a}^{0} i, a < 0 = \sum_{i=0}^{-a}(-i) = -\sum_{i=0}^{-a} i \tag{11}$$

$$\sum_{i=0}^{n}(x_i + y_i)^k = (x_i + y_i)^0 + \sum_{i=1}^{n}(x_i + y_i)^k \tag{12}$$

Proofs

Put one statement per line and justify using names of identities, axioms, previously derived statements, etc.

Identities $(p = q)$ Derive one side from the other. Each line should be a statement of unconditional truth.

Implication $(p \to q)$ Implications are statements that are only guaranteed to be conditionally true. Therefore, we must assume the premise (p) and derive the consequence (q) from the assumption of truth. The statement of assumption is critical because the premises *are not guaranteed to be true*. Note that if a problems asks you to prove p if and only if (iff) q, then this is a bidirectional proof. This means you need to show $p \to q$ and $q \to p$.

- **Direct Proof** $(p \to q)$ Assume p and derive q directly.
- **Proof by Contrapositive** $(\neg q \to \neg p)$ Assume $\neg q$ and derive $\neg p$.
- **Proof by Contradiction** $(p \wedge \neg q)$ Assert the truth of $p \wedge \neg q$ and truthfully derive something false. This falsehood contradicts the initial statement. Since the initial statement is only true when $p \to q$ is false, $p \to q$ must be true.

Induction This proof technique only works when the set you are working over is countable. Prove a basis step. Then state the inductive hypothesis. This is a statement that should be true of an arbitrary element (non-basis) element over the set that on which you're working. Let this element be called the n^{th} element. Assume the truth of this statement and show that it remains true for the $n^{th} + 1$ element.

Counting two ways When proving combinatorial identities, it suffices to show that an identity is equivalent to another expression that is known and accurately counts the size of the set of interest.