CMPSCI 240: Reasoning Under Uncertainty
First Midterm Exam

February 16, 2017.

Name: ______________________________ ID: ____________________

Instructions:

- Answer the questions directly on the exam pages.
- Show all your work for each question since providing more detail including comments and explanations can help with assignment of partial credit.
- If the answer to a question is a number, *unless the problem says otherwise*, you may give your answer using arithmetic operations, such as addition, multiplication, "choose" notation and factorials (e.g., "$9 \times 35! + 2$" or "$0.5 \times 0.3/(0.2 \times 0.5 + 0.9 \times 0.1)$" is fine).
- If you need extra space, use the back of a page.
- No books, notes, calculators or other electronic devices are allowed. Any cheating will result in a grade of 0.
- If you have questions during the exam, raise your hand.

Question	Value	Points Earned
1	10	
2	10	
3	10	
4	10	
5	10	
6	8+2 (Extra Credit)	
Total	58+2 (Extra Credit)	

Question 1. (*10 points*) Indicate whether each of the following statements is TRUE or FALSE. No justification is required.

1.1 (*2 points*): *The probability of the sample space is zero, i.e.,* $P(\Omega) = 0$.

Answer: FALSE.

1.2 (*2 points*): *For any events* A *and* B *with* $P(A) > 0$ *and* $P(B) > 0$ *then* $P(A|B) = P(B|A)$.

Answer: FALSE.

1.3 (*2 points*): *If I roll a fair six-sided dice twice, the probability the second value equals the first value is* $1/6$.

Answer: TRUE.

1.4 (*2 points*): *If I roll a fair six-sided dice three times then the probability of getting three different values is* $\frac{6\times5\times4}{6^3}$.

Answer: TRUE.

1.5 (*2 points*): $\binom{3}{0} + \binom{3}{1} + \binom{3}{2} + \binom{3}{3} = 2^3$. *Remember that* $0! = 1$.

Answer: TRUE.

Question 2. (*10 points*) Every morning I have to wait exactly 1, 2, 3, 4, or 5 minutes for my bus to appear and some waiting times are more likely than others. We can model this as an experiment where the sample space is $\Omega = \{1, 2, 3, 4, 5\}$ and the probability rule satisfies:

$$P(\{1\}) = 1/6 \quad , \quad P(\{2\}) = 1/3 \quad , \quad P(\{3\}) = 1/3 \quad , \quad P(\{4\}) = 1/12 \quad , \quad P(\{5\}) = 1/12$$

Given the information above:

2.1 (*2 points*): *What is the probability of the event where I wait exactly 1 or exactly 2 minutes?*

Answer: $1/6 + 1/3 = 1/2$.

2.2 (*2 points*): *What is the probability of the event where I wait 3 or more minutes?*

Answer: $1/3 + 1/12 + 1/12 = 1/2$.

2.3 (*2 points*): *What's the probability of the event where I wait 5 minutes conditioned on the event that I wait 4 or more minutes?*

Answer: $P(\{5\}|\{4,5\}) = \frac{P(\{5\}\cap\{4,5\})}{P(\{4,5\})} = \frac{1/12}{1/12+1/12} = 1/2$.

2.4 (*2 points*): *What's the probability that my bus is already there when I get to the bus stop?*

Answer: This event is not defined in the probability model. (We also accepted 0 as an answer.)

2.5 (*2 points*): *What is the probability that I wait 5 minutes tomorrow and 5 minutes the next day? You may assume the waiting times on different days is independent.*

Answer: $1/12 \times 1/12 = 1/144$.

Question 3. (*10 points*) Alice, Bob, and Charlie each put their name in a hat. They then each pick a random name (without replacement) from the hat and buy a present for the person whose name they pick. You may assume that Alice picks first, then Bob, then Charlie.

3.1 (*2 points*): *Let $\Omega = \{ABC, ACB, \ldots\}$ be the set of outcomes for the experiment where, for example, "ACB" indicates that "Alice", "Charlie", and "Bob" were the names received by Alice, Bob, and Charlie respectively. Write out all the possible outcomes in Ω.*

Answer: $\Omega = \{ABC, ACB, BAC, BCA, CAB, CBA\}$.

3.2 (*1 points*): *Write the subset of Ω that corresponds to the event that Alice picks her own name.*

Answer: $\{ABC, ACB\}$.

3.3 (*2 points*): *What's the probability that Alice picks her own name?*

Answer: $|\{ABC, ACB\}|/|\Omega| = 2/6 = 1/3$.

3.4 (*2 points*): *What's the probability that everybody picks their own name?*

Answer: $|\{ABC\}|/|\Omega| = 1/6$.

3.5 (*3 points*): *Are the events "Alice picks her own name" and "Bob picks his own name",*

Independent?	True or False
Disjoint?	True or False
Atomic events?	True or False

Question 4. (*10 points*) Your favorite singer has a Twitter account but only 20% of her tweets are written by her whereas 80% of the tweets are written by her assistant. When she writes a tweet there is a 0.5 probability that there is a typo in the tweet. When her assistant writes a tweet there is never a typo. Let S be the event that the singer writes the next tweet and let A be the event that the assistant writes it. Let T be the event that there is a typo in the tweet.

4.1 (*4 points*): *Enter values for the following probabilities:*

$P(S) = 0.2$ $\qquad$ $P(T|S) = 0.5$ $\qquad$ $P(T^c|S) = 0.5$ $\qquad$ $P(T|A) = 0$

4.2 (*2 points*): *What's the probability the tweet is written by the singer and it contains a typo?*

Answer: $0.2 \times 0.5 = 0.1$

4.3 (*2 points*): *What's the probability the tweet does not contain a typo?*

Answer: $0.2 \times 0.5 + 0.8 = 0.9$

4.4 (*2 points*): *If the tweet does not contain a typo, what is the probability it was written by the assistant?*

Answer: $0.8/0.9 = 8/9 = 0.888\ldots$

Question 5. (*10 points*) There are 100 students living in a dorm: 40 live on the first floor, 40 live on the second floor, and 20 live on the third floor. Suppose I write the name of each student on a piece of paper and place all the pieces in a hat. I then randomly pick two names out of the hat (without replacement).

5.1 (*2 points*): *What's the probability they both live on the first floor?*

Answer: $(40/100) \times (39/99) = 26/165 = 0.1576\dots$

5.2 (*2 points*): *What's the probability at least one of them lives on the first floor?*

Answer: $(40/100) \times (39/99) + (40/100) \times (60/99) + (60/100) \times (40/99) = 106/165 = 0.642642\dots\dots$

5.3 (*2 points*): *What's the probability they both live on the same floor?*

Answer: $(40/100) \times (39/99) + (40/100) \times (39/99) + (20/100) \times (19/99) = 35/99 = 0.353535\dots$

5.4 (*2 points*): *What's the probability they live on different floors?*

Answer: $1 - 35/99 = 64/99$

5.5 (*2 points*): *If I picked the names with replacement, what is the probability that both names (which could be the same) live on the first floor?*

Answer: $(40/100)^2 = 4/25 = 0.16$

Question 6. (*10 points*) Suppose a classroom has 100 seats and these are arranged as 10 rows of 10 seats each. Suppose that a class has 50 students enrolled and in each lecture, when a student arrives she is equally likely to pick any seat that is currently unoccupied.

6.1 (*1 points*): *What is the probability the first student that enters the class sits in the left most seat in the front row?*

Answer: $1/100$

6.2 (*2 points*): *What is the probability the second student that enters the class sits in the left most seat in the front row? Hint: You may want to take into account the possibility that the first student may or may not be sitting in that seat.*

Answer: $99/100 \times 1/99 = 1/100$.

6.3 (*1 points*): *What is the probability that all 50 students are seated in the front five rows? Hint: There are $\binom{100}{50}$ different subsets of seats that could end up being occupied and each is equally likely.*

Answer: $1/\binom{100}{50}$.

6.4 (*2 points*): *What is the probability the front row is full?*

Answer: There are $\binom{90}{40}$ ways to seat the 50 students such that the front row is full. Hence, the probability is $\binom{90}{40}/\binom{100}{50} = 1763/2970916$.

6.5 (*2 points*): *What is the probability that every row is either full or empty?*

Answer: There are $\binom{10}{5}$ ways to seat the 50 students such that the every row is full or empty. Hence, the probability is $\binom{10}{5}/\binom{100}{50}$.

6.6 (*2 points*): *Extra Credit: What is the probability that no two students are sat to the immediate left or right of each other?*

Answer: If no two students are sat next to each other there must be exactly 5 students in each row. There are 6 possible configurations for each row

`F-F-F-F-F-` , `-F-F-F-F-F` , `F--F-F-F-F` , `F-F--F-F-F` , `F-F-F--F-F` , `F-F-F-F--F`

so 6^{10} configurations in total. Hence the probability is $6^{10}/\binom{100}{50}$.