Probabilistic Context Free Grammars

Lecture #14

Introduction to Natural Language Processing
CMPSCI 585, Fall 2004

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(including slides from Jason Eisner)

Ambiguity in Parsing

- Time flies like an arrow.
- Fruit flies like a banana.
- I saw the man with the telescope.

How to solve this combinatorial explosion of ambiguity?

1. First try parsing without any weird rules, throwing them in only if needed.
2. Better: every rule has a weight.
   A tree's weight is total weight of all its rules.
   Pick the overall "lightest" parse of sentence.
3. Can we pick the weights automatically?
   We'll get to this later …

CYK Parser

Input: A string of words, grammar in CNF
Output: yes/no
Data structure: n x n table
  rows labeled 0 to n-1, columns 1 to n
  cell (i,j) lists constituents spanning i,j

For each i from 1 to n
  Add to (i-1,i) all Nonterminals that could produce the word at (i-1,i)

time 1 flies 2 like 3 an 4 arrow 5

	1	2	3	4	5
0	NP 3 Vst 3				
1		NP 4 VP 4			
2			P 2 V 5		
3				Det 1	
4					N 8

1 S → NP VP
6 S → Vst NP
2 S → S PP
1 VP → V NP
2 VP → VP PP
1 NP → Det N
2 NP → NP PP
3 NP → NP NP
0 PP → P NP

CYK Parser

For **width** from 2 to n
  For **start** from 0 to n-width
    Define **end** to be **start+width**
    For **mid** from **start+1** to **end-1**
      For every constituent in (**start, mid**)
        For every constituent in (**mid,end**)
          For all ways of combining them (if any)
            Add the resulting constituent to (**start,end**).

time 1 flies 2 like 3 an 4 arrow 5

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1		NP 4 VP 4			
2			P 2 V 5		
3				Det 1	
4					N 8

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1 VP → V NP
2 VP → VP PP
1 NP → Det N
2 NP → NP PP
3 NP → NP NP
0 PP → P NP

time 1 flies 2 like 3 an 4 arrow 5

	1	2	3	4	5
0	NP 3 Vst 3	NP 10			
1		NP 4 VP 4			
2			P 2 V 5		
3				Det 1	
4					N 8

1 S → NP VP
6 S → Vst NP
2 S → S PP
1 VP → V NP
2 VP → VP PP
1 NP → Det N
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time 1 flies 2 like 3 an 4 arrow 5

	1	2	3	4	5
0	NP 3 Vst 3	NP 10 S 8			
1		NP 4 VP 4			
2			P 2 V 5		
3				Det 1	
4					N 8

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time 1 flies 2 like 3 an 4 arrow 5

	1	2	3	4	5
0	NP 3 Vst 3	NP 10 S 8 S 13			
1		NP 4 VP 4			
2			P 2 V 5		
3				Det 1	
4					N 8

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	1	2	3	4	5
0	NP 3 Vst 3	NP 10 S 8 S 13			
1		NP 4 VP 4			
2			P 2 V 5		PP 12
3				Det 1	NP 10
4					N 8

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time 1 flies 2 like 3 an 4 arrow 5

	1	2	3	4	5
0	NP 3 Vst 3	NP 10 S 8 S 13			
1		NP 4 VP 4			
2			P 2 V 5		PP 12 VP 16
3				Det 1	NP 10
4					N 8

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1		NP 4 VP 4			
2			P 2 V 5		PP 12 VP 16
3				Det 1	NP 10
4					N 8

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time 1 flies 2 like 3 an 4 arrow 5

	1	2	3	4	5
0	NP 3 Vst 3	NP 10 S 8 S 13			
1		NP 4 VP 4			NP 18
2			P 2 V 5		PP 12 VP 16
3				Det 1	NP 10
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1		NP 4 VP 4			NP 18 S 21 VP 18
2			P 2 V 5		PP 12 VP 16
3				Det 1	NP 10
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time 1 flies 2 like 3 an 4 arrow 5

0	NP 3 Vst 3	NP 10 S 8 S 13			NP 24
1		NP 4 VP 4			NP 18 S 21 VP 18
2			P 2 V 5		PP 12 VP 16
3				Det 1	NP 10
4					N 8

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1 NP → Det N
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time 1 flies 2 like 3 an 4 arrow 5

0	NP 3 Vst 3	NP 10 S 8 S 13			NP 24 S 22
1		NP 4 VP 4			NP 18 S 21 VP 18
2			P 2 V 5		PP 12 VP 16
3				Det 1	NP 10
4					N 8

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1		NP 4 VP 4			NP 18 S 21 VP 18
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1		NP 4 VP 4			NP 18 S 21 VP 18
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1		NP 4 VP 4			NP 18 S 21 VP 18
2			P 2 V 5		PP 12 VP 16
3				Det 1	NP 10
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	1	2	3	4	5
0	NP 3 Vst 3	NP 10 S 8 S 13			NP 24 S 22 S 27 NP 24 S 27
1		NP 4 VP 4			NP 18 S 21 VP 18
2			P 2 V 5		PP 12 VP 16
3				Det 1	NP 10
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	1	2	3	4	5
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1		NP 4 VP 4			NP 18 S 21 VP 18
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	1	2	3	4	5
0	NP 3 Vst 3	NP 10 S 8 S 13			NP 24 S 22 S 27 NP 24 S 27 S 22 S 27
1		NP 4 VP 4			NP 18 S 21 VP 18
2			P 2 V 5		PP 12 VP 16
3				Det 1	NP 10
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2 VP → VP PP
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Follow backpointers ...

S

time 1 flies 2 like 3 an 4 arrow 5

	1	2	3	4	5
0	NP 3 Vst 3	NP 10 S 8 S 13			NP 24 S 22 S 27 NP 24 S 27 S 22 S 27
1		NP 4 VP 4			NP 18 S 21 VP 18
2			P 2 V 5		PP 12 VP 16
3				Det 1	NP 10
4					N 8

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time 1 flies 2 like 3 an 4 arrow 5

	1	2	3	4	5
0	NP 3 Vst 3	NP 10 S 8 S 13			NP 24 S 22 S 27 NP 24 S 27 S 22 S 27
1		NP 4 VP 4			NP 18 S 21 VP 18
2			P 2 V 5		PP 12 VP 16
3				Det 1	NP 10
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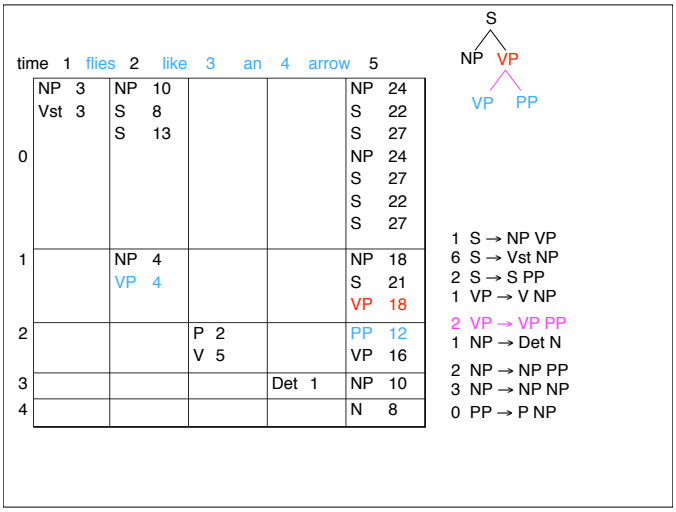
time 1 flies 2 like 3 an 4 arrow 5
0
NP 3
Vst 3
NP 10
S 8
S 13
NP 24
S 22
S 27
NP 24
S 27
S 22
S 27
1
NP 4
VP 4
NP 18
S 21
VP 18
2
P 2
V 5
PP 12
VP 16
3
Det 1
NP 10
4
N 8
S
NP VP
VP PP
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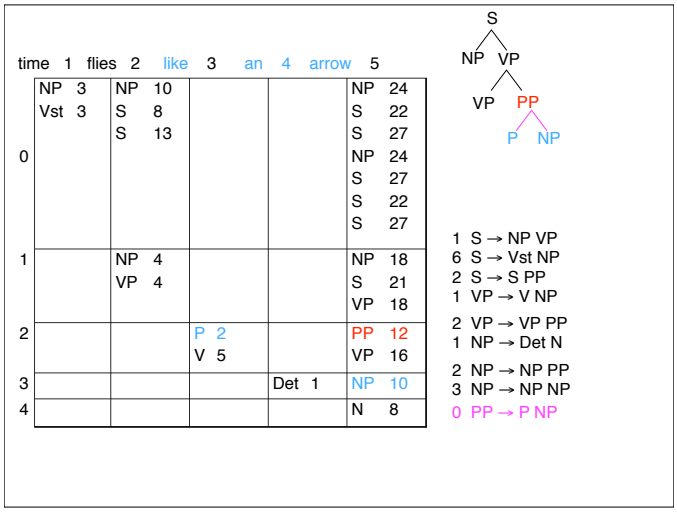
time 1 flies 2 like 3 an 4 arrow 5
0
NP 3
Vst 3
NP 10
S 8
S 13
NP 24
S 22
S 27
NP 24
S 27
S 22
S 27
1
NP 4
VP 4
NP 18
S 21
VP 18
2
P 2
V 5
PP 12
VP 16
3
Det 1
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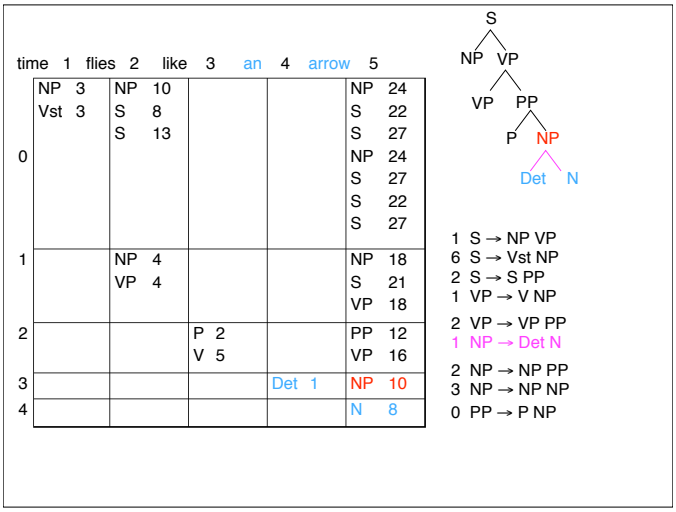
time 1 flies 2 like 3 an 4 arrow 5
0
NP 3
Vst 3
NP 10
S 8
S 13
NP 24
S 22
S 27
NP 24
S 27
S 22
S 27
1
NP 4
VP 4
NP 18
S 21
VP 18
2
P 2
V 5
PP 12
VP 16
3
Det 1
NP 10
4
N 8
S
NP VP
VP PP
P NP
Det N
1 S → NP VP
6 S → Vst NP
2 S → S PP
1 VP → V NP
2 VP → VP PP
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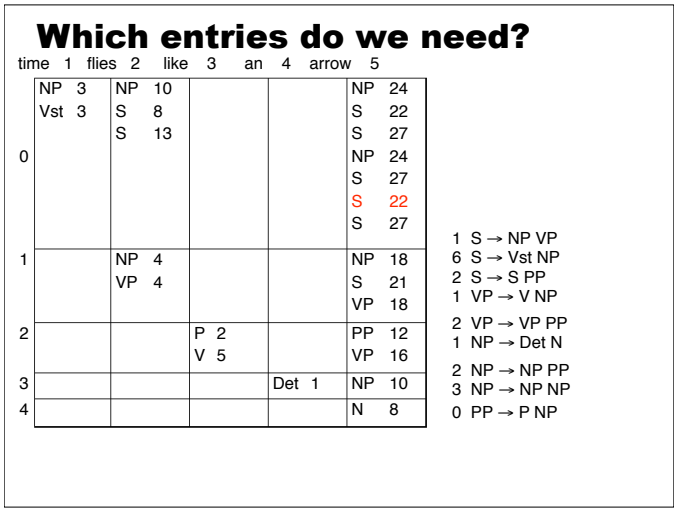
Which entries do we need?
time 1 flies 2 like 3 an 4 arrow 5
0
NP 3
Vst 3
NP 10
S 8
S 13
NP 24
S 22
S 27
NP 24
S 27
S 22
S 27
1
NP 4
VP 4
NP 18
S 21
VP 18
2
P 2
V 5
PP 12
VP 16
3
Det 1
NP 10
4
N 8
1 S → NP VP
6 S → Vst NP
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2 VP → VP PP
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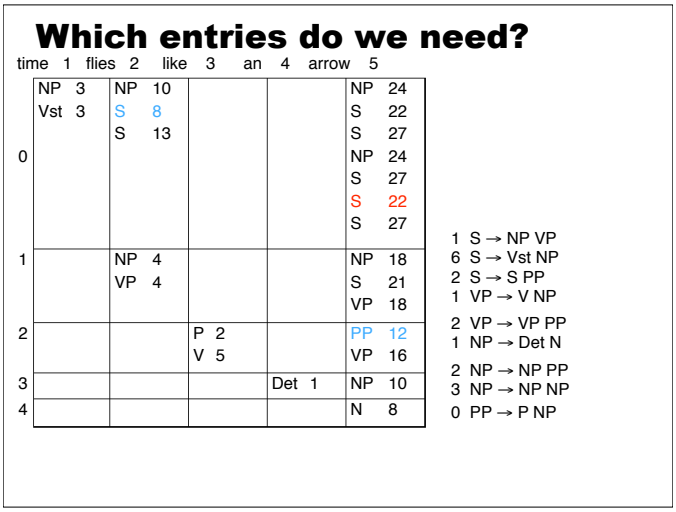
Which entries do we need?
time 1 flies 2 like 3 an 4 arrow 5
0
NP 3
Vst 3
NP 10
S 8
S 13
NP 24
S 22
S 27
NP 24
S 27
S 22
S 27
1
NP 4
VP 4
NP 18
S 21
VP 18
2
P 2
V 5
PP 12
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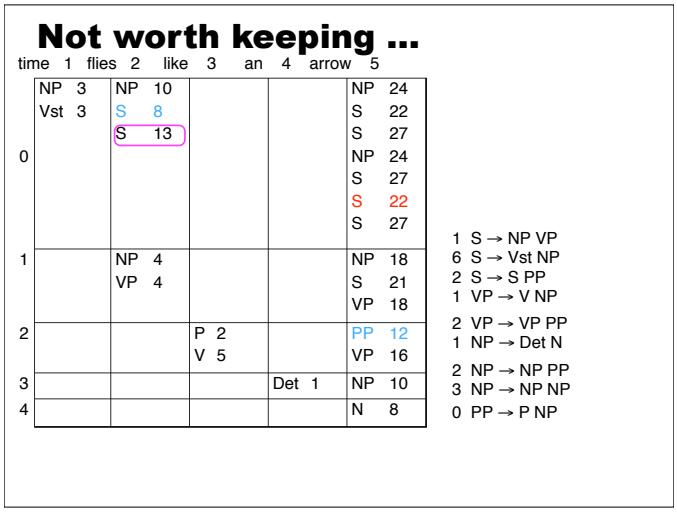
Not worth keeping ...
time 1 flies 2 like 3 an 4 arrow 5
0
NP 3
Vst 3
NP 10
S 8
S 13
NP 24
S 22
S 27
NP 24
S 27
S 22
S 27
1
NP 4
VP 4
NP 18
S 21
VP 18
2
P 2
V 5
PP 12
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… since it just breeds worse options

	time 1	flies 2	like 3	an 4	arrow 5
0	NP 3 Vst 3	NP 10 S 8 S 13			NP 24 S 22 S 27 NP 24 S 27 S 22 S 27
1		NP 4 VP 4			NP 18 S 21 VP 18
2			P 2 V 5		PP 12 VP 16
3				Det 1	NP 10
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Keep only best-in-class!

	time 1	flies 2	like 3	an 4	arrow 5
0	NP 3 Vst 3	NP 10 S 8 S 13	inferior stock		NP 24 S 22 S 27 NP 24 S 27 S 22 S 27
1		NP 4 VP 4			NP 18 S 21 VP 18
2			P 2 V 5		PP 12 VP 16
3				Det 1	NP 10
4					N 8

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Keep only best-in-class!
(and backpointers so you can recover parse)

	time 1	flies 2	like 3	an 4	arrow 5
	NP 3 Vst 3	NP 10 S 8			NP 24 S 22
1		NP 4 VP 4			NP 18 S 21 VP 18
2			P 2 V 5		PP 12 VP 16
3				Det 1	NP 10
4					N 8

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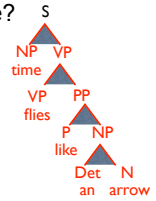
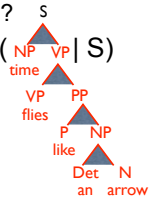
Probabilistic Trees

- Instead of lightest weight tree, take highest probability tree
- Given any tree, your assignment generator would have some probability of producing it!
- Just like using n-grams to choose among strings …
- What is the probability of this tree?



Probabilistic Trees

- Instead of lightest weight tree, take highest probability tree
- Given any tree, your assignment generator would have some probability of producing it!
- Just like using n-grams to choose among strings …
- What is the probability of this tree?
-
- You rolled a lot of independent dice...

P( | S)

Chain rule: One word at a time

p(time flies like an arrow)
= p(time)
* p(flies | time)
* p(like | time flies)
* p(an | time flies like)
* p(arrow | time flies like an)

Chain rule + backoff (to get trigram model)

p(time flies like an arrow)
= p(time)
* p(flies | time)
* p(like | time flies)
* p(an | time flies like)
* p(arrow | time flies like an)

Chain rule – written differently

p(time flies like an arrow)
= p(time)
* p(time flies | time)
* p(time flies like | time flies)
* p(time flies like an | time flies like)
* p(time flies like an arrow | time flies like an)

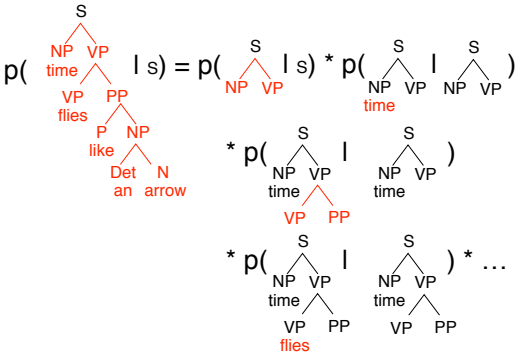
Proof: p(x,y | x) = p(x | x) * p(y | x, x) = 1 * p(y | x)

Chain rule + backoff

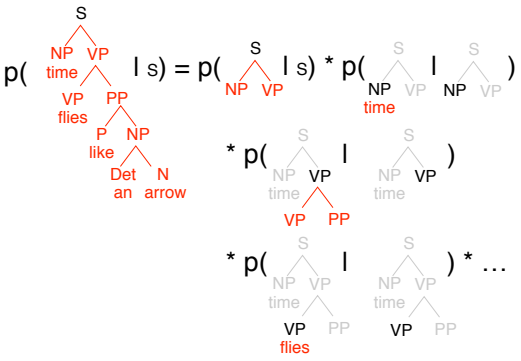
p(time flies like an arrow)
= p(time)
* p(time flies | time)
* p(time flies like | time flies)
* p(time flies like an | time flies like)
* p(time flies like an arrow | time flies like an)

Proof: p(x,y | x) = p(x | x) * p(y | x, x) = 1 * p(y | x)

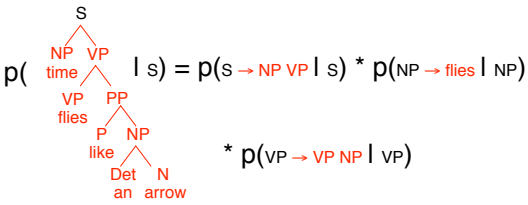
Chain rule: One node at a time



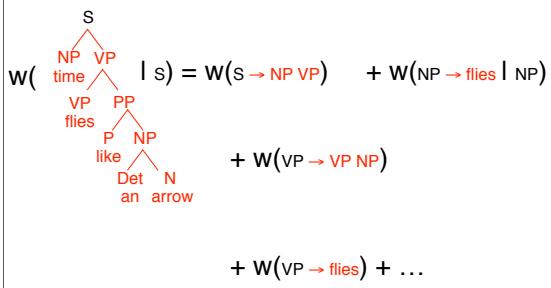
Chain rule + backoff



Simplified notation

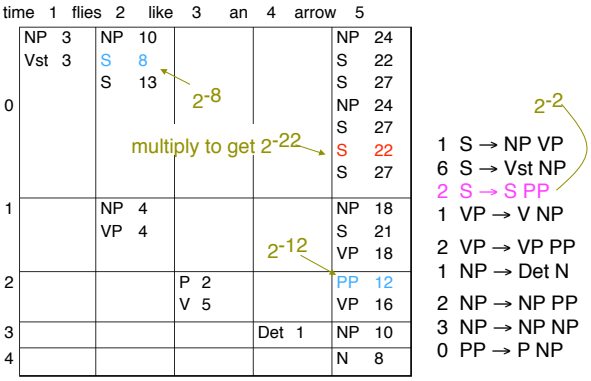


Already have a CKY alg for weights …

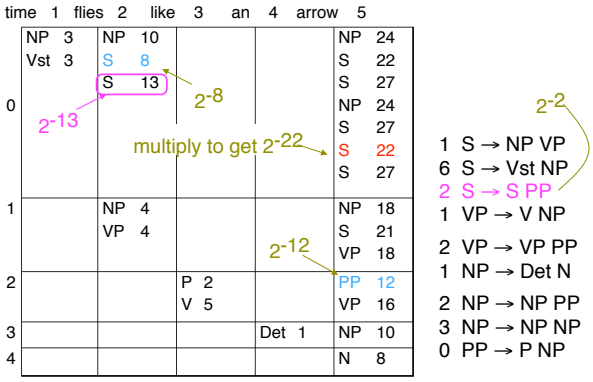


Just let w(X → Y Z) = -log p(X → Y Z | X)

Then lightest tree has highest prob 49



Need only best-in-class to get best parse



Why probabilities not weights?

- We just saw probabilities are really just a special case of weights …
- … *but* we can estimate them from training data by counting and smoothing! Use all of our lovely probability theory machinery!

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Probabilistic Context Free Grammars (PCFGs)

A PCFG G consists of the usual parts of a CFG

- A set of terminals, $\{w^k\}, k = 1, \ldots, V$
- A set of nonterminals, $\{N^i\}, i = 1, \ldots, n$
- A designated start symbol, N^1
- A set of rules, $\{N^i \rightarrow \zeta^j\}$, (where ζ^j is a sequence of terminals and nonterminals)

and

- A corresponding set of probabilities on rules such that:

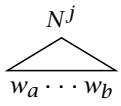
$$\forall i \quad \sum_j P(N^i \rightarrow \zeta^j) = 1$$

PCFG notation

Sentence: sequence of words $w_1 \cdots w_m$

w_{ab}: the subsequence $w_a \cdots w_b$

N^i_{ab}: nonterminal N^i dominates $w_a \cdots w_b$



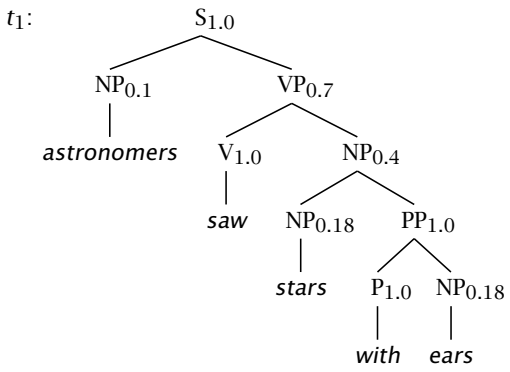
$N^i \stackrel{*}{\Rightarrow} \zeta$: Repeated derivation from N^i gives ζ.

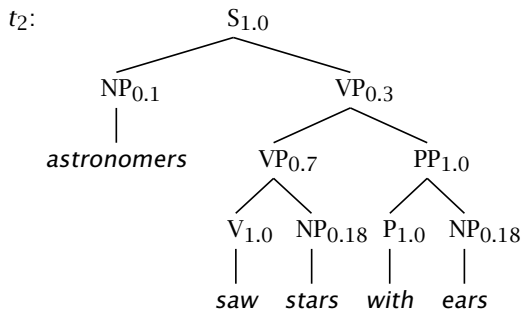
PCFG probability of a string

$$P(w_{1n}) = \sum_t P(w_{1n}, t) \quad t \text{ a parse of } w_{1n}$$
$$= \sum_{\{t:\text{yield}(t)=w_{1n}\}} P(t)$$

A simple PCFG (in CNF)

S → NP VP	1.0	NP → NP PP	0.4
PP → P NP	1.0	NP → *astronomers*	0.1
VP → V NP	0.7	NP → *ears*	0.18
VP → VP PP	0.3	NP → *saw*	0.04
P → *with*	1.0	NP → *stars*	0.18
V → *saw*	1.0	NP → *telescopes*	0.1





The two parse trees' probabilities and the sentence probability

$$P(t_1) = 1.0 \times 0.1 \times 0.7 \times 1.0 \times 0.4 \times 0.18 \times 1.0 \times 1.0 \times 0.18$$
$$= 0.0009072$$
$$P(t_2) = 1.0 \times 0.1 \times 0.3 \times 0.7 \times 1.0 \times 0.18 \times 1.0 \times 1.0 \times 0.18$$
$$= 0.0006804$$
$$P(w_{15}) = P(t_1) + P(t_2) = 0.0015876$$

Assumptions of PCFGs

1. Place invariance (like time invariance in HMM):

   $\forall k \quad P(N^j_{k(k+c)} \rightarrow \zeta)$ is the same

2. Context-free:

   $P(N^j_{kl} \rightarrow \zeta | \text{words outside } w_k \ldots w_l) = P(N^j_{kl} \rightarrow \zeta)$

3. Ancestor-free:

   $P(N^j_{kl} \rightarrow \zeta | \text{ancestor nodes of } N^j_{kl}) = P(N^j_{kl} \rightarrow \zeta)$

The sufficient statistics of a PCFG are thus simply counts of how often different local tree configurations occurred (= counts of which grammar rules were applied).

Some features of PCFGs

Reasons to use a PCFG, and some idea of their limitations:

- Partial solution for grammar ambiguity: a PCFG gives some idea of the plausibility of a sentence.
- But, in the simple case, not a very good idea, as independence assumptions are two strong (e.g., not lexicalized).
- Gives a probabilistic language model for English.
- In the simple case, a PCFG is a worse language model for English than a trigram model.
- Better for grammar induction (Gold 1967 vs. Horning 1969)
- Robustness. (Admit everything with low probability.)

Some features of PCFGs

- A PCFG encodes certain biases, e.g., that smaller trees are normally more probable.
- One can hope to combine the strengths of a PCFG and a trigram model.

We'll look at simple PCFGs first. They have certain inadequacies, but we'll see that most of the state-of-the-art probabilistic parsers are fundamentally PCFG models, just with various enrichments to the grammar

A slightly different task

- Been asking: What is probability of generating a given *tree*?
  - To pick tree with highest prob: useful in parsing.
-
- But could also ask: What is probability of generating a given *string* with the generator?
  - To pick string with highest prob: useful in speech recognition, as substitute for an n-gram model.
    - ("Put the file in the folder" vs. "Put the file and the folder")
  - To get prob of generating string, must add up probabilities of all trees for the string …

Could just add up the parse probabilities

time 1 flies 2 like 3 an 4 arrow 5

0	NP 3 Vst 3	NP 10 S 8 S 13		2^{-22} 2^{-27} 2^{-27} 2^{-22} 2^{-27}	NP 24 S 22 S 27 NP 24 S 27 S 22 S 27
1		NP 4 VP 4			NP 18 S 21 VP 18
2			P 2 V 5		PP 12 VP 16
3				Det 1	NP 10
4					N 8

oops, back to finding exponentially many parses

1 S → NP VP
6 S → Vst NP
2 S → S PP
1 VP → V NP
2 VP → VP PP
1 NP → Det N
2 NP → NP PP
3 NP → NP NP
0 PP → P NP

Any more efficient way?

time 1 flies 2 like 3 an 4 arrow 5

0	NP 3 Vst 3	NP 10 S 2^{-8} S 2^{-13}			NP 24 S 22 S 27 NP 24 S 27 S 2^{-22} S 2^{-27}
1		NP 4 VP 4			NP 18 S 21 VP 18
2			P 2 V 5		PP 2^{-12} VP 16
3				Det 1	NP 10
4					N 8

1 S → NP VP
6 S → Vst NP
2^{-2} S → S PP
1 VP → V NP
2 VP → VP PP
1 NP → Det N
2 NP → NP PP
3 NP → NP NP
0 PP → P NP

Add as we go … (the "inside algorithm")

time 1 flies 2 like 3 an 4 arrow 5

0	NP 3 Vst 3	NP 10 S $2^{-8}+2^{-13}$			NP 24 S 22 S 27 NP 24 S 27 S 2^{-22} $+2^{-27}$
1		NP 4 VP 4			NP 18 S 21 VP 18
2			P 2 V 5		PP 2^{-12} VP 16
3				Det 1	NP 10
4					N 8

1 S → NP VP
6 S → Vst NP
2^{-2} S → S PP
1 VP → V NP
2 VP → VP PP
1 NP → Det N
2 NP → NP PP
3 NP → NP NP
0 PP → P NP

Add as we go … (the "inside algorithm")

	time 1	flies 2	like 3	an 4	arrow 5
0	NP 3 Vst 3	NP 10 S $2^{-8}+2^{-13}$			NP 2^{-22} $+2^{-27}$ S 2^{-22} $+2^{-27}$ $+2^{-27}$ $+2^{-22}$ $+2^{-27}$
1		NP 4 VP 4			NP 18 S 21 VP 18
2			P 2 V 5		PP 2^{-12} VP 16
3				Det 1	NP 10
4					N 8

1 S → NP VP
6 S → Vst NP
2^{-2} S → S PP
1 VP → V NP
2 VP → VP PP
1 NP → Det N
2 NP → NP PP
3 NP → NP NP
0 PP → P NP

Inside and Outside Probabilities

Probability of all possible rule re-writes for generating words inside position p to q, given that non-terminal j exactly spans p to q.

$$\text{Inside} = \beta_j(p,q) = P(w_{pq}|N^j_{pq}, G)$$

Probability of all possible rule re-writes for generating words *outside* position p to q, *and* that non-terminal j exactly spans p to q.

$$\text{Outside} = \alpha_j(p,q) = P(w_{1(p-1)}, N^j_{pq}, w_{(q+1)m}|G)$$

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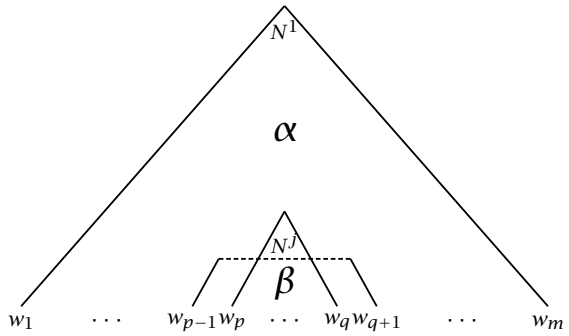
Inside and Outside Probabilities



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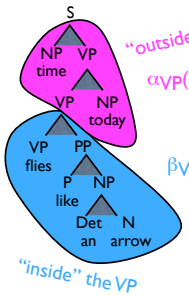
Inside & Outside Probabilities



"outside" the VP

$\alpha_{VP}(1,5)$ = p(time VP today | S)

$\beta_{VP}(1,5)$ = p(flies like an arrow | VP)

$\alpha_{VP}(1,5) * \beta_{VP}(1,5)$
= p(time [VP flies like an arrow] today | S)

"inside" the VP

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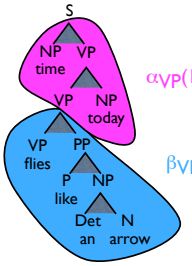
Inside & Outside Probabilities



$\alpha_{VP}(1,5)$ = p(time VP today | S)

$\beta_{VP}(1,5)$ = p(flies like an arrow | VP)

$\alpha_{VP}(1,5) * \beta_{VP}(1,5)$ / $\beta_S(0,6)$

$$= \frac{p(\text{time flies like an arrow today \& VP(1,5)} \mid S)}{p(\text{time flies like an arrow today} \mid S)}$$

$$= p(\text{VP(1,5)} \mid \text{time flies like an arrow today}, S)$$

So $\alpha_{VP}(1,5) * \beta_{VP}(1,5)$ / $\beta_S(0,6)$
is probability that there is a VP here, given all of the observed data (words)

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Probability of a string

Inside probability

$$P(w_{1m}|G) = P(N^1 \Rightarrow w_{1m}|G)$$
$$= P(w_{1m}, N^1_{1m}, G) = \beta_1(1,m)$$

Base case: We want to find $\beta_j(k,k)$ (the probability of a rule $N^j \rightarrow w_k$):

$$\beta_j(k,k) = P(w_k|N^j_{kk}, G)$$
$$= P(N^j \rightarrow w_k|G)$$

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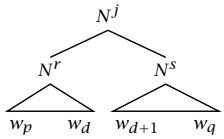
Probability of a string

Induction: We want to find $\beta_j(p,q)$, for $p < q$. As this is the inductive step using a Chomsky Normal Form grammar, the first rule must be of the form $N^j \rightarrow N^r \quad N^s$, so we can proceed by induction, dividing the string in two in various places and summing the result:



These inside probabilities can be calculated bottom up.

For all j,

$$\begin{aligned}
\beta_j(p,q) &= P(w_{pq}|N^j_{pq},G) \\
&= \sum_{r,s}\sum_{d=p}^{q-1} P(w_{pd},N^r_{pd},w_{(d+1)q},N^s_{(d+1)q}|N^j_{pq},G) \\
&= \sum_{r,s}\sum_{d=p}^{q-1} P(N^r_{pd},N^s_{(d+1)q}|N^j_{pq},G) \\
&\quad P(w_{pd}|N^j_{pq},N^r_{pd},N^s_{(d+1)q},G) \\
&\quad P(w_{(d+1)q}|N^j_{pq},N^r_{pd},N^s_{(d+1)q},w_{pd},G) \\
&= \sum_{r,s}\sum_{d=p}^{q-1} P(N^r_{pd},N^s_{(d+1)q}|N^j_{pq},G) \\
&\quad P(w_{pd}|N^r_{pd},G)P(w_{(d+1)q}|N^s_{(d+1)q},G) \\
&= \sum_{r,s}\sum_{d=p}^{q-1} P(N^j \rightarrow N^r N^s)\beta_r(p,d)\beta_s(d+1,q)
\end{aligned}$$

Inside probabilities as CYK

	1	2	3	4	5
1	β_{NP} = 0.1		β_S = 0.0126		β_S = 0.0015876
2		β_{NP} = 0.04 β_V = 1.0	β_{VP} = 0.126		β_{VP} = 0.015876
3			β_{NP} = 0.18		β_{NP} = 0.01296
4				β_P = 1.0	β_{PP} = 0.18
5					β_{NP} = 0.18
	astronomers	*saw*	*stars*	*with*	*ears*

Outside probabilities

Probability of a string: For any k, $1 \le k \le m$,

$$\begin{aligned}
P(w_{1m}|G) &= \sum_j P(w_{1(k-1)},w_k,w_{(k+1)m},N^j_{kk}|G) \\
&= \sum_j P(w_{1(k-1)},N^j_{kk},w_{(k+1)m}|G) \\
&\quad \times P(w_k|w_{1(k-1)},N^j_{kk},w_{(k+1)n},G) \\
&= \sum_j \alpha_j(k,k)P(N^j \rightarrow w_k)
\end{aligned}$$

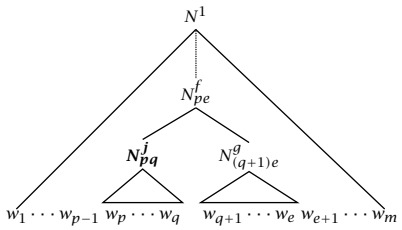
Inductive (DP) calculation: One calculates the outside probabilities top down (after determining the inside probabilities).

Outside probabilities

Base Case:

$\alpha_1(1,m) = 1$
$\alpha_j(1,m) = 0$, for $j \neq 1$

Inductive Case: it's either a left or right branch – we will some over both possibilities and calculate using outside *and* inside probabilities



Outside probabilities, Inductive case

$$\begin{aligned}
\alpha_j(p,q) &= \Big[\sum_{f,g}\sum_{e=q+1}^{m} P(w_{1(p-1)},w_{(q+1)m},N^f_{pe},N^j_{pq},N^g_{(q+1)e})\Big] \\
&\quad + \Big[\sum_{f,g}\sum_{e=1}^{p-1} P(w_{1(p-1)},w_{(q+1)m},N^f_{eq},N^g_{e(p-1)},N^j_{pq})\Big] \\
&= \Big[\sum_{f,g}\sum_{e=q+1}^{m} P(w_{1(p-1)},w_{(e+1)m},N^f_{pe})P(N^j_{pq},N^g_{(q+1)e}|N^f_{pe}) \\
&\quad \times P(w_{(q+1)e}|N^g_{(q+1)e})\Big] + \Big[\sum_{f,g}\sum_{e=1}^{p-1} P(w_{1(e-1)},w_{(q+1)m},N^f_{eq}) \\
&\quad \times P(N^g_{e(p-1)},N^j_{pq}|N^f_{eq})P(w_{e(p-1)}|N^g_{e(p-1)})\Big] \\
&= \Big[\sum_{f,g}\sum_{e=q+1}^{m} \alpha_f(p,e)P(N^f \rightarrow N^j\ N^g)\beta_g(q+1,e)\Big] \\
&\quad + \Big[\sum_{f,g}\sum_{e=1}^{p-1} \alpha_f(e,q)P(N^f \rightarrow N^g\ N^j)\beta_g(e,p-1)\Big]
\end{aligned}$$

Probability that a rule is used

As with a HMM, we can form a product of the inside and outside probabilities. This time:

$$\begin{aligned}&\alpha_j(p,q)\beta_j(p,q)\\&= P(w_{1(p-1)}, N^j_{pq}, w_{(q+1)m}|G)P(w_{pq}|N^j_{pq}, G)\\&= P(w_{1m}, N^j_{pq}|G)\end{aligned}$$

$$P(N^j_{pq}|w_{1m}, G) = \frac{P(N^j_{pq}|w_{1m}, G)}{P(w_{1m}|G)} = \frac{\alpha_j(p,q)\beta_j(p,q)}{\beta_1(1,m)}$$

This is an "expected count" for the number of times this rule occurred.

Overall probability of a node existing

As with a HMM, we can form a product of the inside and outside probabilities. This time:

$$\begin{aligned}&\alpha_j(p,q)\beta_j(p,q)\\&= P(w_{1(p-1)}, N^j_{pq}, w_{(q+1)m}|G)P(w_{pq}|N^j_{pq}, G)\\&= P(w_{1m}, N^j_{pq}|G)\end{aligned}$$

Therefore,

$$p(w_{1m}, N_{pq}|G) = \sum_j \alpha_j(p,q)\beta_j(p,q)$$

Just in the cases of the root node and the preterminals, we know there will always be some such constituent.

Learning PCFGs (1)

- We would like to calculate how often each rule is used:

$$\hat{P}(N^j \rightarrow \zeta) = \frac{C(N^j \rightarrow \zeta)}{\sum_\gamma C(N^j \rightarrow \gamma)}$$

- If we have labeled data, we count and find out
- Relative frequency again gives maximum likelihood probability estimates
- This is the motivation for building *Treebanks* of hand-parsed sentences

Learning PCFGs (2) Inside-Outside

- Otherwise we work iteratively from expectations of current model.
- We construct an EM training algorithm, as for HMMs
- For each sentence, at each iteration, we work out expectation of how often each rule is used using inside and outside probabilities
- We assume sentences are independent and sum expectations over parses of each
- We re-estimate rules based on these 'counts'