COMPSCI H311 Spring 2020 - Homework 1

Released: Wednesday 29 January 2020. Due: Tuesday 4 February in class

List your collaborators if any. The writeup you present must be your own work in presentation, and you must acknowledge all sources of aid other than course staff and course material.

1. Path-based SCC

Gabow (2000)[1] (p.2 right) gives a generic algorithm for path-based strongly connected components. In bullet 2, "contract the cycle" means that the nodes are now considered equivalent. This equivalence class might increase later (in Fig. 1, node 6 is added to [2, 4, 5]).

What is the complexity of this generic algorithm if using a Union-Find data structure for equivalence classes? (You might use any of the versions discussed in class).

2. Stack growing and shrinking

In class, we saw that a stack (array) can be extended in amortized constant time if we double the size when it is full. If we pop elements and the stack shrinks significantly below its allocated size, we'd like to shrink it to use less memory. (Assume the memory allocator may need to move the part still used somewhere else). Propose a resizing condition (actual size is some fraction of the allocated size) and a new size so that the amortized cost of any sequence of push and pop operations is still constant-time.

[1] `https://www.cs.princeton.edu/courses/archive/spr09/cos423/Lectures/bi-gabow.pdf`