COMPSCI 501: Formal Language Theory

Lecture 2: Deterministic Finite Automata

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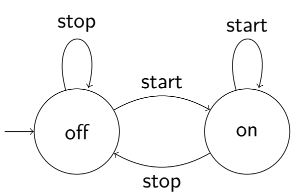
Proposed Revised Grading

- **Homework**: 36% (six homeworks)
- **Moodle quizzes**: 4% (throughout semester)
- **Midterm 1**: 20% (Thu Feb 21, 7 pm, ILC S131)
- **Midterm 2**: 20% (Wed Apr 10, 7 pm, ILC S131)
- **Final**: 20% (Thu May 9, 10:30 am, Goessmann 20)

Automata: the simplest computers

Switch: just one bit of memory



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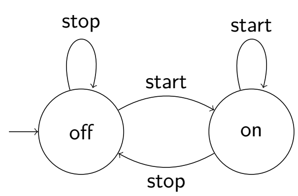


What happens if we press "stop" in "off" state?
Should describe behavior *completely*

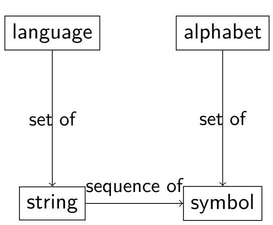
Strings and Languages

Alphabet (usually Σ): any nonempty finite set

String: *finite* sequence of symbols from the alphabet

Σ^*: set of all (finite) strings over Σ, incl. *empty string* (denoted ϵ).

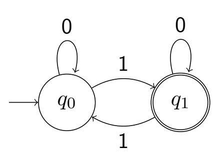
A **language** is an arbitrary subset of strings (of Σ^*)



Decision problem for a language X: for a given input string $w \in \Sigma^*$, answer whether $w \in X$

Parity: Even or Odd ?

We'll use automata to *recognize languages*.



Bit strings (0 or 1) with an odd number of ones.

q_0: initial state; q_1: accepting state

Formal Definition

A (deterministic) finite automaton is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$:

- Q is a finite set of *states*
- Σ is a finite *alphabet* (of input *symbols*)
- $\delta : Q \times \Sigma \to Q$ is the *transition function*
- $q_0 \in Q$ is the *start state*
- $F \subseteq Q$ is the *set of accept states*.

Language of a DFA

Informally: M accepts string w if on input w it ends up in an accepting state.

More precisely: let $w = w_1 w_2 \dots w_n$. Then M accepts w if there is a sequence of states $r_0, r_1, \dots r_n$ from Q such that

- $r_0 = q_0$ (initial state)
- $\delta(r_i, w_{i+1}) = r_{i+1}$ (transitions on symbols from string)
- $r_n \in F$ (accept state)

$L(M)$ (language of M) = set of strings accepted by M

Def. A language is called a **regular language** if some finite automaton recognizes it.

Some Simple Patterns

Strings that

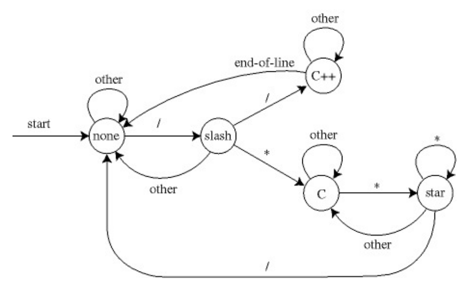
- *start* with a given string
  if not, go to "dead state" (will never accept)
- *end* with a given string
  in general, must "remember" last k symbols (pattern length)
- *contain* a given string (pattern search)
  Knuth-Morris-Pratt algorithm constructs DFA for pattern

contains/ends with pattern: easy via NFA - DFA construction

Automata for a finite set of strings:
{"they", "are", "more" }
{"this", "is", "his" }

Recognizing Comments in C / C++

Two types of comments:
/* anything */
// anything newline

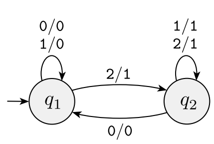


Which states should be accepting?
Source file should not end inside comment

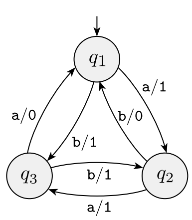
Figure: D. Eppstein

(Finite State) Transducers

Outputs a string (one output symbol for each input)
a.k.a. Mealy machines



T_1



T_2

Change in formal definition:

- output alphabet Γ
- transition function: $\delta : Q \times \Sigma \to Q \times \Gamma$

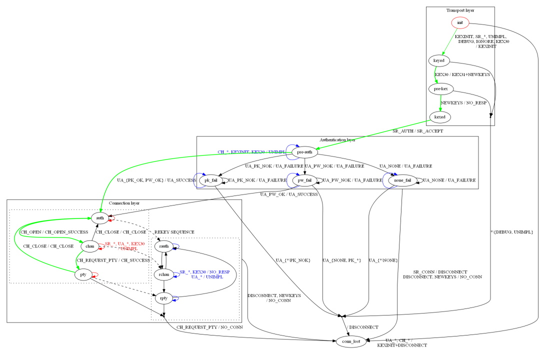
Finite State Machines for Testing

Some Fundamental Testing problems:

- Determine the state after a test (homing/distinguishing sequence)
- Verify that M is in a given state s (state verification)
- Conformance testing: Given M (black-box) and a FSM S (specification), determine whether M is equivalent to S
- Machine identification: identify unknown black-box machine M

Automata Learning

Learn a FSM model of a black-box system
for protocols, security, legacy software



Model for SSH (Fiterau-Brostean & Vaandrager)

Regular Operations on Languages

If A and B are languages, we define:

Union: $A \cup B = \{x \mid w \in A$ or $x \in B\}$

Concatenation: $A \circ B = \{xy | x \in A$ and $y \in B\}$

(Kleene) Star: $A^* = \{x_1 x_2 \ldots x_k \mid k \geq 0$ and $x_i \in A\}$

We'll see that regular languages are closed under each of these operations.

Product Construction for Union

Let $M_1 = (Q_1, \Sigma, \delta_1, q_1^0, F_1)$ and $M_2 = (Q_2, \Sigma, \delta_2, q_2^0, F_2)$

Is string w accepted by M_1 *or* M_2?

Can only read string once $\Rightarrow$ run both automata in parallel
know state of M_1 *and* M_2 at each point: pair of states

$\Rightarrow$ state space is *cartesian product* $Q_1 \times Q_2$

- $Q = Q_1 \times Q_2$
- Σ is the same (common) alphabet
- $\delta((r_1, r_2), a) = (\delta(r_1, a), \delta(r_2, a))$
- $q_0 = (q_1^0, q_2^0)$
- $F = \{(r_1, r_2) \mid r_1 \in F_1$ or $r_2 \in F_2\}$
  at least one automaton accepts

Other Language Operations

- *Intersection*
  product (like for union), but $F = F_1 \times F_2$ (both must accept)
- *Complement*
  same automaton structure, swap accept/non-accept states
- *Concatenation*
  must split string w into $x \in L(M_1)$ and $y \in L(M_2)$: where?
  construction will use *nondeterminism*