COMPSCI 501 Spring 2019 - Homework 5

Released 4/1/2019 Due in Gradescope on Fri Apr 12, 11:59pm

Instructions.

You may work in groups, but you must write solutions yourself. List your collaborators.

All your answers should contain a rigorous argument of correctness.

Please submit a PDF file. You may submit a scanned handwritten document, but a typed submission is preferred. Please assign pages to questions in Gradescope.

Q1 (5p). Let *SPARSE-SAT* be the problem: given a formula in CNF form and an integer k, find a satisfying assignment in which at most k variables are TRUE, if one exists. Prove that *SPARSE-SAT* is NP-complete.

Q2 (10p). Consider the problem of determining, given an undirected graph $G = (V, E)$, whether G contains a clique of size $\lfloor V/2 \rfloor$. Is this problem in NP? Is it NP-complete? Prove your statements.

Q3 (15p). Consider the following problem, called *THIRD-FRACTION*: given a set $\{a_1, \ldots, a_n\}$ of positive integers, decide whether we can find indices $i_1, \ldots, i_k$ such that $\sum_j a_{i_j} = \frac{1}{3} \sum_j a_j$.
Show that *THIRD-FRACTION* is NP-complete.

Q4 (5p). For $k \geq 1$, let *k-COLOR* be
$\{G \mid G$ is an undirected graph whose vertices can be properly colored with at most k colors $\}$.
For which values of k is the problem *k-COLOR* in P? For which values is it NP-complete?
Prove your statements.

Q5 (10p). Are all NP-hard problems necessarily PSPACE-complete? What about the converse?

Q6 (15p). Suppose T is a set of "tile types", $T = \{t_1, t_2, \ldots, t_k\}$, and H and V be two relations $H, V \subseteq (T \times T)$, with $H(t_i, t_j)$ meaning t_j can be placed immediately to the right of t_i, and $V(t_i, t_j)$ meaning t_j can be placed immediately below t_i. Given an integer n, let R be a length-n horizontal row of tiles (possibly with repeats) from T satisfying all H-constraints. Consider the language *EXTEND-TILING* $= \{\langle n, T, H, V, R \rangle \mid$ there exists a tiling of the $n \times n$ grid satisfying all H and V constraints, with R as its topmost row $\}$. Show that *EXTEND-TILING* is NP-complete.