

COMPSCI 501 Spring 2019 - Homework 3

Released 2/25/2019

Due in Gradescope on Thu Mar 7, 11:59pm

Instructions.

You may work in groups, but you must write solutions yourself. List your collaborators.

All your answers should contain a rigorous argument of correctness.

Please submit a PDF file. You may submit a scanned handwritten document, but a typed submission is preferred. Please assign pages to questions in Gradescope.

Q1 (15p). We say some language L *isolates* languages L_1 and L_2 if $L_1 \subseteq L$ and $L_2 \subseteq \bar{L}$. Suppose L_1 and L_2 are disjoint co-Turing-recognizable languages. Prove that L_1 and L_2 are isolated by some *decidable* language.

Q2 (5+5p). Prove that a language is decidable if and only if it is enumerable in lexicographic order. Prove that every infinite Turing-recognizable language has an infinite decidable subset.

Q3 (5p). Let $S = \{\langle M \rangle \mid M \text{ is a DFA that accepts } w^R \text{ whenever it accepts } w\}$. Show that S is decidable.

Q4 (15p). Let $L = \{\langle M, Q \rangle \mid M \text{ is a TM, } Q \text{ is a state of } M, \text{ and there is some input } w \text{ whose computation in } M \text{ reaches the state } Q\}$. Prove or disprove: L is undecidable.

Q5 (10p). Show that determining whether a CFG generates all strings in 1^* is decidable.

In other words, show that $\{\langle G \rangle \mid G \text{ is a CFG over } \{0, 1\} \text{ and } 1^* \subseteq L(G)\}$ is a decidable language.

Q6 (5p). Goldbach's Conjecture states that every even integer greater than 2 is expressible as the sum of two prime numbers. Show that if HALT were decidable, then we would be able to determine whether Goldbach's Conjecture were true or false.