COMPSCI 501 Spring 2019 - Homework 2

Released 2/12/2019 Due in Gradescope on Fri Feb 22, 11:59pm

Instructions.
You may work in groups, but you must write solutions yourself. List your collaborators.
All your answers should contain a rigorous argument of correctness.
Please submit a PDF file. You may submit a scanned handwritten document, but a typed submission is preferred. Please assign pages to questions in Gradescope.

Q1 (15p). Let $A/B = \{w | wx \in A \text{ for some } x \in B\}$. Show that if A is context free and B is regular, then A/B is context free.
Q2 (10p).

a) Is the language $L = \{a^n b^{n^2} | n \geq 0\}$ a CFL? Prove or disprove.
b) Give an example to show that context free languages are not closed under intersection.

Q3 (5p). Design an unambigous CFG G with terminals $X = \{0, 1, (,), |, *, \varnothing, e\}$ that generates exactly the regular expressions over $\{0, 1\}$. Here, e denotes the empty string in a regular expression. The usual precedence rules apply: first unary star, then concatenation, then union.
Q4 (15p). CFGs for the following languages:

a) $\{$ w | in every prefix of w the number of a's is at least the number of b's $\}$
b) $\{$ w | the number of a's and the number of b's in w are equal $\}$
c) $\{$ w | the number of a's is at least the number of b's in w $\}$

Extra credit for *unambiguous* grammars (a:1p + b:2p + c:2p).
Q5 (10p). A *substitution* is a mapping s of every symbol of an alphabet $a \in \Sigma$ to some language $s(a) = L_a$ (possibly over a different alphabet). We can extend s to strings (mapping every letter): $s(\varepsilon) = \varepsilon$, $s(wa) = s(w)s(a)$, and then to languages: $s(L) = \cup_{w \in L} s(w)$.
Show that if L is a CFL and s is a substitution such that $s(a)$ is a CFL for every symbol $s \in \Sigma$, then $s(L)$ is also a CFL.
Q6 (5p). Let CFG G be the following grammar: $S \to aSb|bY|Ya$ $\quad Y \to bY|aY|\varepsilon$
Give a simple description of G in English. Use that description to give a CFG for $\overline{L(G)}$, the complement of $L(G)$.