COMPSCI 501 Spring 2019 - Homework 1

Released 1/26/2019   Due in Gradescope on Thu Feb 7, 11:59pm

Instructions.
You may work in groups, but you must write solutions yourself. List your collaborators.
All your answers should contain a rigorous argument of correctness.
Please submit a PDF file. You may submit a scanned handwritten document, but a typed submission is preferred. Please assign pages to questions in Gradescope.

Q1 (15p). Give a DFA and a regular expression for each of the following languages:

(a) The language L_1 of all strings over $\{x, y, z\}$ where the first character is *unique* (e.g., $yxxzxxz \in L_1$, but $zyyxyzyx \notin L_1$).

(b) The language L_2 of all strings over $\{0, 1\}$ *except* for 11 and 111.

(c) The language of all strings over $\{0, 1\}$ containing 000 as a substring but not 111.

Q2 (10p). What is the language decided by the following DFA?
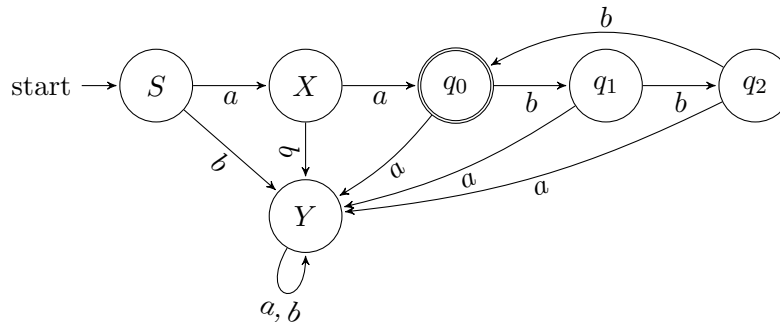
Q3 (5p). Prove or provide a counterexample for the following statement: By flipping accept and non-accept states in an NFA, we obtain an NFA which accepts the language's complement.

Q4 (15p). For languages L_1 and L_2, define the language $weave(L_1, L_2)$ as

$$weave(L_1, L_2) = \{x_1 y_1 x_2 y_2 \ldots x_n y_n \mid x_1 x_2 \ldots x_n \in L_1 \text{ and } y_1 y_2 \ldots y_n \in L_2\}.$$

For instance, if $ABC \in L_1$ and $789 \in L_2$, then $A7B8C9 \in weave(L_1, L_2)$. Prove or give a counterexample: if L_1 and L_2 are regular, so is $weave(L_1, L_2)$.

Q5 (10p). Prove that the language $L = \{pqp \mid p, q \in \{0, 1\}^+\}$ is not regular.

Q6 (5p). Let L be the language over the alphabet $\{0, 1, 2\}$ consisting of strings that use at most two of the three characters in the alphabet (so L includes 011101010 and 222, but not 0100102). Give a regular expression corresponding to this language.